Weak and Strong Taylor methods for numerical solutions of stochastic differential equations

Maria Siopacha and Josef Teichmann

Department of Mathematical Methods in Economics, Vienna University of Technology, Wiedner Hauptstrasse 8–10/105–1, A-1040 Vienna, Austria.
{siopacha, josef.teichmann}@fam.tuwien.ac.at

ABSTRACT

We work on results of Malliavin-Thalmaier-Watanabe for strong and weak Taylor expansions of solutions of perturbed stochastic differential equations (SDEs). We are interested in SDEs depending on a parameter ϵ, such that at $\epsilon = 0$ the model is reduced to a well-known, simple model. In Fournié et al. [1] methods to deal with such situations have been developed for elliptic models. Our approach is built in the hypoelliptic framework. Similar expansions are described in Malliavin and Thalmaier [2].

In particular, we work out weight expressions for the Taylor coefficients of the expansion. Our main theorem is the existence of weak Taylor approximation schemes of order $n \geq 0$ under appropriate integrability assumptions and a hypoellipticity assumption.

The results are applied to LIBOR market models in order to deal with the typical stochastic drift and with stochastic volatility. In contrast to other accurate methods like numerical schemes for the full SDE, we obtain easily tractable expressions for accurate pricing. The results are underlined by some numerical simulations.

References

[1] Fournié E., Lebuchoux J. and Touzi N., *Small Noise Expansion and Importance Sampling*, Asymptotic Analysis, pp. 361–376, 1997.

[2] Malliavin. P and Thalmaier A., *Stochastic Calculus of Variations in Mathematical Finance* Springer, 2006.