Option Pricing in Illiquid Markets and Nonlinear Black-Scholes Equations

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Abstract. We study properties of option prices in illiquid markets. As a starting point, we introduce various models for illiquid markets such as the model of Cetin, Jarrow and Protter (2004) or the model of Frey(2000) and show that the problem of hedging European options leads to closely related fully nonlinear versions of the standard Black-Scholes PDE. The main part is devoted to a study of analytical properties of this PDE. In particular, we show that under natural conditions the option prices provided by the model (the solutions of the nonlinear PDE) have the properties of a convex (but non-coherent) risk measure. Moreover, we study asymptotic properties of solutions as the market depth tends to zero. The necessary technical tools are comparison and stability theorems for viscosity solutions.