Competing players in illiquid markets: predatory trading vs. liquidity provision
Preliminary draft version

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Abstract

We consider a multi player situation in an illiquid market in which one player tries to liquidate a large portfolio in a short time span, while other players know of the seller's intention and try to make a profit by trading in this market over a longer time horizon. We show that the illiquidity characteristics of the market determine the optimal strategy for these players: They either provide liquidity to the seller, or they prey on him by simultaneous selling. If they provide liquidity, it might be sensible for the seller to pre-announce a trading intention ("sunshine trading").

We show that if the number of informed players is large, liquidity provision is the optimal strategy irrespective of the market environment. Furthermore, the market price immediately drops after the start of trading and exhibits no drift thereafter, i.e., the market acts efficiently as long as the number of informed players is large.

1 Introduction

A variety of circumstances such as a margin call or a stop-loss strategy in combination with a large price drop can force a market participant (the "seller") to sell a large asset position urgently. In other situations, it is necessary to buy a large asset position quickly, for example in a short squeeze. From now on, we will assume that the market participant needs to sell a large asset position; equivalent statements hold for the buying situation.

In all of the mentioned situations, the seller must achieve a trading target in a fixed and relatively short time horizon; e.g., a margin call has to be covered by the end of the day. Other market participants (the "predators") might have obtained information on the seller's situation and could try to earn a profit by exploiting it. The predators typically do not face the same time constraint the seller is facing; they can afford to maintain a long or a short position for a number of days. Probably the most famous example of such a situation is the alleged trading against the hedge fund LTCM that was covered extensively by public media and academic research (see e.g., [Low01] and [Cai03]).

In order to capture the structure of this situation, we develop a two stage model of an illiquid market. In the first stage, the seller as well as the predators trade; in the second stage, only the predators trade and unwind the asset positions they acquired during the first stage. Our market model incorporates illiquidity by applying a permanent as well as a temporary impact in a similar fashion to the market model proposed by Almgren and Chriss [AC01].

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We derive a Nash equilibrium of optimal trading strategies for the seller and the predators. The predators' optimal strategy depends heavily on the type of illiquidity exhibited in the market. If the major obstacle is to attract counterparties *in a limited time frame* and therefore the temporary impact is high compared to the permanent impact (a "truly illiquid market"), then the optimal strategy for the predators is to cooperate with the seller: they should buy some of the seller's assets and sell them at a later point in time. On the other hand, if the permanent impact outweighs the temporary impact, i.e., it is expensive to find counterparties *at all* (a "nervous market"), the optimal behavior of the predators is the opposite: they should sell in parallel to the seller and buy back at a later point in time. In this case, the price is pushed far down during the first stage and recovers during the second stage, resulting in price overshooting. This effect disappears as the number of predators increases; for a large number of predators, the market price incorporates the seller's intentions almost instantly and exhibits little drift thereafter. Therefore, the weak form of the efficient market hypothesis does hold in our model, but the strong one does not.

We find that in truly illiquid markets, the seller[1] achieves a higher return when predators are participating than when he is selling by himself. Therefore, pre-announcing his trade ("sunshine trading") appears to be sensible in such a market. In a nervous market, the seller's return is reduced by predators; however, as the number of predators increases, the optimal strategy for the predators changes from predation to cooperation and the return for the seller increases again, potentially even above the return received without any predators.

Our research builds on previous work in three research areas. Extensive literature has empirically investigated the market impact of large transactions (e.g., [KS72], [BHS95], [CL95], [Mön04]). The results of this work, most notably the identification of temporary and permanent impact, have led to theoretical models of illiquid markets. We apply a market model similar to the one proposed by Almgren and Chriss [AC01]. Several alternative models have been proposed (e.g., [Alm03], [OW06]); the advantages and disadvantages of these models are still a topic of ongoing research.

A second line of research focuses on the interaction of market participants in an illiquid market. Brunnermeier and Pedersen [BP05] showed that predatory trading and price overshooting occur when the total rate of trading as well as the asset positions of all traders face exogenous constraints. In their model, providing liquidity to a large seller is never optimal, irrespective of the market environment. As a side effect of the trading constraint, their model market is inefficient: even if the number of informed predators is large, the market price changes continuously in reaction to the trading of the seller and the predators. In a market model similar to the one applied by us, Carlin et al. [CLV05] show that predative trading occurs, but their model cannot explain price overshooting. To explain cooperative behavior, they assume that all market participants repeatedly need to execute large transactions; in such a framework, predation can be punished by applying a tit-for-tat strategy.

In a third line of research, the effects of sunshine trading are investigated. In a theoretical investigation, Admati and Pfleiderer [AP91] find that sunshine trading is always increasing the seller's return as long as speculators do not face market entry costs. The empirical evidence on the benefit of trade pre-announcements appears to be mixed (see e.g., [Har97], [DP04]).

The remainder of this paper is structured as follows. In Section 2, we introduce the market model and specify the game theoretic optimization problem. As a preparation for the general two stage model, we investigate a one stage model in Section 3. In this model, the seller and the predators face the same time constraint, i.e., the predators do not have the opportunity to trade after the seller finished selling. In the main Section 4, we turn to the more general two stage framework and derive our main results. After identifying the Nash equilibrium of optimal trading strategies in Subsection 4.1, we investigate the qualitative properties of our model in three example markets in Subsection 4.2. Thereafter, we summarize the general properties in Subsection 4.3. Section 5 concludes. All proofs are given in the Appendix.

[1] For reasons of brevity, we refer to the seller as "him" instead of the longer "her or him".

2 The market model

We follow the notation used in [CLV05]. The market consists of a risk free asset and a risky asset. Trading takes place in continuous time. We assume that the risk free asset does not generate interest. In this market we consider $n+1$ strategic players and a number of noise traders. We denote the time-dependent risky asset positions of the strategic players by $X_0(t)$, $X_1(t)$, ..., $X_n(t)$ and assume that they are differentiable in t. Their trading $\dot{X}_i(t)$ affects the market price in the form of a permanent impact and a temporary impact. Trades at time t are thus executed at the price

$$P(t) = \tilde{P}(t) + \gamma \sum_{i=0}^{n} (X_i(t) - X_i(0)) + \lambda \sum_{i=0}^{n} \dot{X}_i(t).$$

Here, $\tilde{P}(t)$ is a one-dimensional arithmetic Brownian motion without drift, starting at $\tilde{P}(0) = P_0$ and defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The second term on the right hand side represents the permanent price impact resulting from the change in total asset position of all strategic players. Its magnitude is determined by the parameter $\gamma > 0$. The third term reflects the temporary impact caused by the net trading speed of all strategic investors. Its magnitude is controlled by the parameter $\lambda > 0$. This price dynamics model is a multi player extension of the framework introduced by Almgren and Chriss [AC01] with linear permanent and linear temporary impact.

In this market, the players are facing the following optimization problem. Each player i knows all other players' initial asset positions $X_j(0)$ and their target asset positions $X_j(T)$ for some fixed point $T > 0$ in the future. We assume that these trading targets are binding; players are not allowed to violate their targets. As in [BP05] and [CLV05], we assume that all players are risk neutral. Therefore, players want to maximize their own *expected* return by choosing an optimal trading strategy $X_i(t)$ given their boundary constraints on $X_i(0)$ and $X_i(T)$. In mathematical terms, each player is looking for a strategy that realizes the maximum

$$\begin{aligned} R_i \quad &:= \quad \max_{X_i} \mathbb{E}(\text{Return for player } i) = \max_{X_i} \mathbb{E}\left(\int_0^T (-\dot{X}_i(t)) P(t) dt \right) \\ &= \quad \max_{X_i} \mathbb{E}\left(-\int_0^T \dot{X}_i(t) \left(\tilde{P}(t) + \gamma \sum_{j=0}^{n} (X_j(t) - X_j(0)) + \lambda \sum_{j=0}^{n} \dot{X}_j(t) \right) dt \right). \end{aligned}$$

Although in principle the strategies X_i might be predictable, we limit our discussion to deterministic strategies, where the function X_i does not depend on the stochastic price component $\tilde{P}(t)$. Hence,

$$R_i \quad = \quad \max_{X_i} \left(-\int_0^T \dot{X}_i(t) \left(P_0 + \gamma \sum_{j=0}^{n} (X_j(t) - X_j(0)) + \lambda \sum_{j=0}^{n} \dot{X}_j(t) \right) dt \right). \tag{1}$$

A set of strategies $(X_0, X_1, ..., X_n)$ satisfying Equation (1) for all $i = 0, 1, ..., n$ constitutes a Nash equilibrium; we call such a set of strategies *optimal*. These are determined by the *expected* price

$$\bar{P}(t) = \mathbb{E}(P(t)) = P_0 + \gamma \sum_{i=0}^{n} (X_i(t) - X_i(0)) + \lambda \sum_{i=0}^{n} \dot{X}_i(t).$$

Whenever we refer to *price* or *return* in the following, we will always refer to the *expected price* $\bar{P}(t)$ and the *expected return* $-\int \dot{X}_i(t) \bar{P}(t) dt$.

3 The one stage model

In this section, we will investigate the optimal strategies in a one stage framework: all players trade over the same time interval $[0, T_1]$. The results in this section will be used in the analysis of

Parameter	Value
Asset position X_0	1
Initial price P_0	10
Duration T_1	1
Permanent impact sensitivity γ	3
Temporary impact sensitivity λ	1

Table 1: Parameter values used for numerical computation of the figures in Section 3.

a two stage model in the following sections in which all players trade in the time interval $[0, T_1]$, but the predators are allowed to continue trading over an extended time interval $[0, T_2]$.

The optimal strategies in the one stage framework were derived by Carlin, Lobo and Viswanathan. We repeat their result from [CLV05]:

Theorem 1 ([CLV05])**.** *Assume that $n+1$ players are trading simultaneously in a time period $t \in [0, T_1]$. They start with asset positions $X_i(0)$ and need to achieve a target asset position $X_i(T_1)$. Furthermore, these players are risk neutral and are aware of all other players' asset positions and trading targets. Then the unique optimal strategies for these $n+1$ players (in the sense of a Nash equilibrium) are given by:*

$$\dot{X}_i(t) = a e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t} + b_i e^{\frac{\gamma}{\lambda}t}$$

with

$$\begin{aligned} a &= \frac{n}{n+2}\frac{\gamma}{\lambda}\left(1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}\right)^{-1} \frac{\sum_{i=0}^{n}(X_i(T_1) - X_i(0))}{n+1} \\ b_i &= \frac{\gamma}{\lambda}\left(e^{\frac{\gamma}{\lambda}T_1} - 1\right)^{-1}\left(X_i(T_1) - X_i(0) - \frac{\sum_{j=0}^{n}(X_j(T_1) - X_j(0))}{n+1}\right). \end{aligned}$$

Proof. See [CLV05]. □

For the rest of this section, we consider the following more specific situation: One player (say player 0) wants to sell an asset position $X_0(0) = X_0$ in the time $[0, T_1)$, i.e. the target is given by $X_0(T_1) = 0$. All other players (i.e., players 1, 2, ..., n) do not want to change their initial and terminal asset positions (for simplicity, we assume that $X_i(0) = X_i(T_1) = 0$ for $i \neq 0$), but they want to exploit their knowledge of player 0's sales.

The result is *preying* of the n players on the first player (see Figure 1 and 2; see Table 1 for the parameter values used for the figures): while the first player is starting to sell off his asset position, the other players sell short the asset and realize a comparatively high price per share; at the end of the trading period, the first player is selling the rest of his asset position for a fairly low price, while the other players are buying back the shares they initially sold short. Since the price has dropped, the preying players need to spend less on average for buying back than they received for initially selling short. In the following, we will refer to player 0 as the "seller" and to the players 1, 2, ..., n as the "predators".

Let us analyze the market prices resulting from the combined trading activities of the seller and the predators in more detail. In Figure 3, we observe that when trading commences in $t = 0$, the expected price jumps downward from its level $\bar{P}(0-) = P_0$ to $\bar{P}(0)$ due to the temporary impact of the selling. After the initial price jump, the expected price $\bar{P}(t)$ is exhibiting a downward trend. The following proposition generalizes this observation.

Proposition 2. *The initial price jump $|\bar{P}(0) - \bar{P}(0-)|$ is an increasing function of the number n of predators, while the drift $\dot{\bar{P}}(t)$ is a decreasing function of n. In the limit case $n \to \infty$, the expected market price instantaneously jumps to*

$$P_0 - \frac{\gamma}{1 - e^{-\frac{\gamma T_1}{\lambda}}} X_0$$

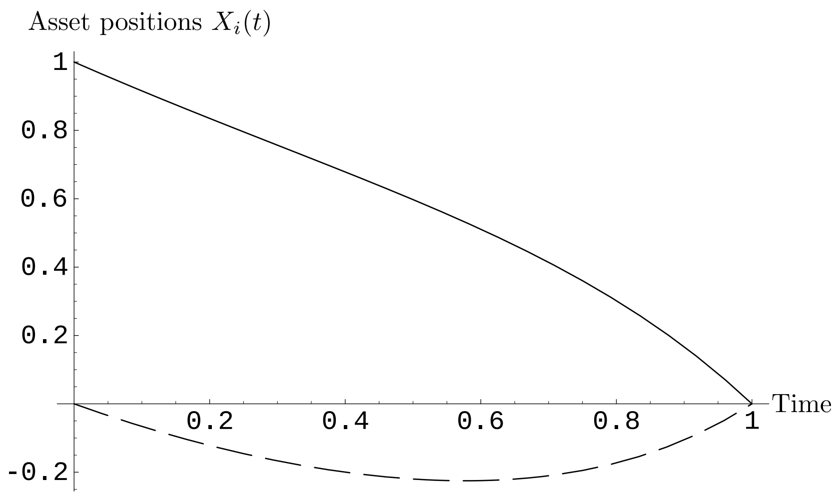

Figure 1: Asset positions $X_i(t)$ over time. The solid line represents the seller, the dashed line the predator ($n = 1$).

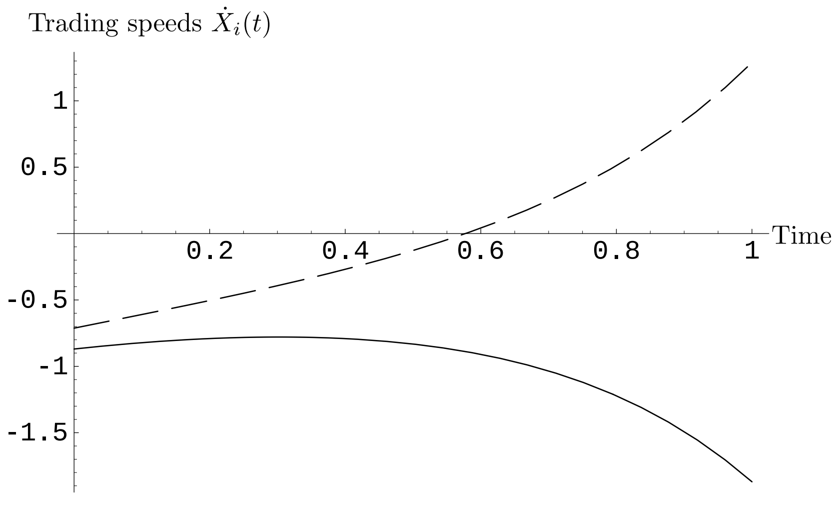

Figure 2: Trading speeds $\dot{X}_i(t)$ over time. The solid line represents the seller, the dashed line the predator ($n = 1$).

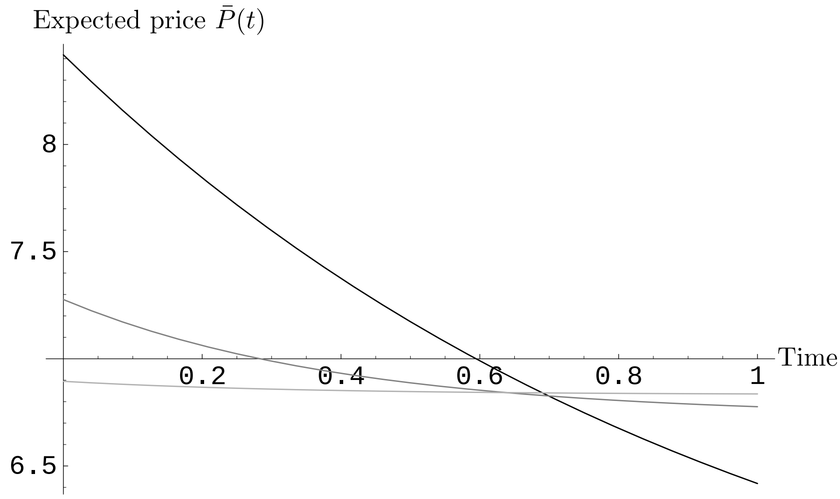

Figure 3: Expected price $\bar{P}(t)$ over time $t \in [0, T_1]$ depending on the number of predators n. The black line corresponds to $n = 1$, the dark grey line to $n = 10$ and the light grey line to $n = 100$. A significant reduction in price drift and an increase in initial price jump from $\bar{P}(0-) = 10$ to $\bar{P}(0)$ can be observed.

and is constant from there on until the end of trading at time $t = T_1$.

This proposition indicates that our model market fulfills the semi-strong form of the efficient markets hypothesis: If relevant information (in our case the seller's intentions) is known by a *sufficiently large* number of market participants, this information is instantaneously fully reflected in market prices. Public information can thus not be used to predict price changes. However, our model market does not fulfill the following version of the strong form of the efficient markets hypothesis; if relevant information is shared by only a *small* number of market participants, then this information is only slowly reflected in market prices.

We will now consider the time after T_1, i.e., after the seller and the predators have stopped trading. The temporary impact of the trades during $[0, T_1]$ vanishes immediately; therefore, only the permanent impact remains. The seller sold X_0 while the predators did not change their asset positions, therefore we obtain an expected market price of $\bar{P}(T_1+) = P_0 - \gamma X_0$ for the time after T_1. If during the trading phase $[0, T_1]$ the price drops below $\bar{P}(T_1+)$, i.e.,

$$\min_{t \in [0, T_1]} \bar{P}(t) - \bar{P}(T_1+) < 0$$

we say that the price *overshoots*. We can now describe the relationship between price overshooting and predatory activity.

Proposition 3. *Without any predators (i.e., nobody is aware of the seller's intentions), the price overshoots by $\lambda X_0 / T_1$. If predators are present, the price overshooting is reduced to*

$$\frac{n}{n+2} \gamma X_0 \frac{e^{-\frac{n}{n+2}\frac{\gamma}{\lambda} T_1}}{1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda} T_1}}$$

which is a decreasing function of the number n of predators.

It is interesting to compare the two models introduced by Brunnermeier and Pedersen [BP05] and by Carlin, Lobo and Viswanathan [CLV05]. Predatory trading occurs in both models: In Brunnermeier and Pedersen's model, an exogenous trading speed constraint makes predation attractive, while in the model by Carlin et al., the continuous price deterioration caused by the seller provides the incentive. Preying introduces price overshooting in the first framework, but it

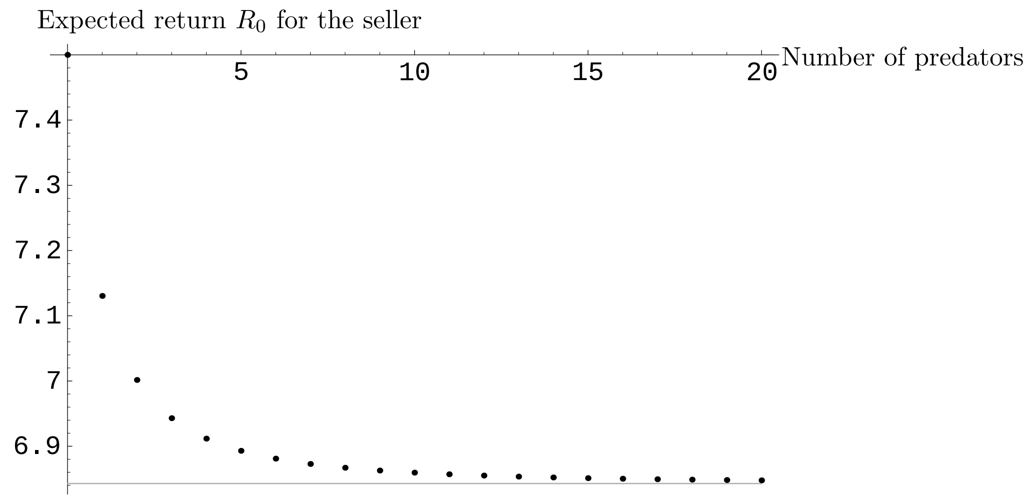

Figure 4: Expected cash return for the seller (player 0) from selling X_0 shares, depending on the number n of predators. The expected return in absence of predators is 7.5 (intersection point of x- and y-axes). The grey line at the bottom corresponds to the limit $n \to \infty$.

reduces price overshooting in the latter. In both models, price overshooting is reduced by additional predators (assuming that at least one predator is active). In our two stage extension of the model of Carlin et al., we will show that depending on the relative importance of permanent and temporary impact, predatory trading can both increase or decrease price overshooting.

In the one stage model considered so far, there is no room for cooperation; preying **always** occurs. This is inconsistent with the real financial market, where liquidity provision and preying coexist. Furthermore, in the one stage model presented above the seller's return is further deteriorating as the number of predators increases; preying becomes more competitive with more players being involved (see Figure 4). In the next section, we will see that these issues disappear in the two stage model.

We conclude this section by providing explicit formulas for the return that the seller can expect to receive.

Proposition 4. *By selling the asset position X_0 during the time $[0, T_1]$, the seller receives a total average cash position of*

$$R_0 = X_0 \left(P_0 - \gamma X_0 \frac{A_2 n^2 + A_1 n + A_0}{B_2 n^2 + B_1 n + B_0} \right)$$

with

$$\begin{array}{lll} A_2 = q(p-1) & A_1 = 3pq - p - 2q & A_0 = (p-1)(q-1) \\ B_2 = (p-1)(q-1) & B_1 = 3(p-1)(q-1) & B_0 = 2(p-1)(q-1) \end{array}$$

and

$$p = e^{\frac{\gamma T_1}{\lambda} \frac{n}{n+2}}, \quad q = e^{\frac{\gamma T_1}{\lambda}}.$$

For $n \to \infty$, R_0 converges to

$$\lim_{n\to\infty} R_0 = X_0 \left(P_0 - \gamma X_0 \frac{1}{1 - e^{-\frac{\gamma}{\lambda} T_1}} \right).$$

Note that the cash received in the limit case $n \to \infty$ is exactly the initial asset position multiplied by the limit of the expected market price derived in Proposition 2.

4 The two-stage model

In the previous section, we have assumed that the seller and the predators are limited to trade during the same time interval. As we have mentioned earlier, in reality the seller is facing a stricter time constraint than the predators do. While the seller usually needs to liquidate his asset position within a few hours, the predators can often afford to unwind their positions at a later point in time. In the following, we therefore extend the one stage model considered so far to a two stage framework and assume that:

- In stage 1, *all* players (the seller and the predators) are trading.
- In stage 2, *only* the predators are trading; the seller is not active.

The first stage runs from $t = 0$ to T_1, the second stage from T_1 to T_2[2]. The two stages thus have durations T_1 respectively $T_2 - T_1$; the asset position of player i is denoted by $X_{i,1}(t)$ with $t \in [0, T_1]$ in stage 1 and $X_{i,2}(t)$ with $t \in [0, T_2 - T_1]$ in stage 2. No trading may occur between the two stages, i.e., $X_{i,1}(T_1) = X_{i,2}(0)$.

The market prices are governed by

$$P_l(t) = \tilde{P}_l(t) + \gamma \sum_{i=0}^{n} (X_{i,l}(t) - X_{i,1}(0)) + \lambda \sum_{i=0}^{n} \dot{X}_{i,l}(t)$$

for $l \in \{1, 2\}$ and $t \in [0, T_l]$. Again, $\tilde{P}_l(t)$ is an arithmetic Brownian motion without drift, starting at $\tilde{P}_1(0) = P_0$ respectively $\tilde{P}_2(0) = \tilde{P}_1(T_1)$.

The seller (player 0) is assumed to liquidate an asset position $X_0 = X_{0,1}(0)$ during stage 1: $X_{0,1}(T_1) = X_{0,2}(t) = 0$ for all $t \in [0, T_2 - T_1]$. The n predators are assumed to have zero asset positions at the beginning of stage 1 and at the end of stage 2: $X_{i,1}(0) = X_{i,2}(T_2 - T_1) = 0$. There are no a-priori restrictions on their asset positions $X_{i,1}(T_1)$ at the end of stage 1. They can be positive, i.e., the predators buy some of the seller's shares in stage 1 and thereby provide liquidity to the seller. Alternatively, they can be negative, i.e., the predators sell parallel to the seller, driving the market price further down and preying on the seller. In this section, we will show that the occurrence of liquidity provision or predation depends on the market characteristics, in particular on the balance between temporary and permanent impact.

We again restrict our analysis to risk neutral players following deterministic strategies. By *price* and *return*, we will refer to the expected prices

$$\bar{P}_l(t) = \mathbb{E}(P_l(t)) = P_0 + \gamma \sum_{i=0}^{n} (X_{i,l}(t) - X_{i,1}(0)) + \lambda \sum_{i=0}^{n} \dot{X}_{i,l}(t)dt$$

and the expected return

$$-\int_0^{T_1} \dot{X}_{i,1}(t)\bar{P}_1(t)dt - \int_0^{T_2 - T_1} \dot{X}_{i,2}(t)\bar{P}_2(t)dt.$$

The maximum return achievable by player i will be denoted by

$$R_i = \max_{X_i}(\text{Expected return for player } i).$$

The remainder of this section is structured in three subsections: In Subsection 4.1, we identify the optimal strategies for the seller and the predators. In Subsection 4.2, we discuss the properties of these strategies in three explicit examples. In Subsection 4.3, we conclude by providing formal theorems stating these properties in our general setting.

[2]Our framework can easily be extended to the case where the second stage runs from $\tilde{T}_1 > T_1$ to T_2

4.1 Optimal strategies

In this subsection we describe the optimal behavior of all $n+1$ players. If the optimal asset positions $X_{i,1}(T_1)$ of the predators at the end of stage 1 are known, the entire optimal strategies are determined by Theorem 1: In stage 1, $n+1$ players are trading and the initial and final asset positions are known; in stage 2, n players are trading and again the initial and final asset positions are known[3]. Therefore, we only need to derive the optimal asset positions $X_{i,1}(T_1)$ for all predators $i = 1, 2, ..., n$.

Theorem 5. *In the unique Nash equilibrium, all predators acquire the same asset position during stage 1:*

$$X_{i,1}(T_1) = -\frac{A_2 n^2 + A_1 n + A_0}{B_3 n^3 + B_2 n^2 + B_1 n + B_0} X_0.$$

The coefficients A_i and B_i are functions of n that converge in the limit $n \to \infty$ (see the Appendix for explicit formulas).

For the special case $n = 1$, we obtain

$$X_{1,1}(T_1) = -\frac{\Big(-2 - e^{\frac{\gamma T_1}{3\lambda}} - e^{\frac{2\gamma T_1}{3\lambda}} + e^{\frac{\gamma T_1}{\lambda}}\Big)\gamma(T_2 - T_1)}{6\Big(-1 + e^{\frac{\gamma T_1}{\lambda}}\Big)\lambda + \Big(2 + e^{\frac{\gamma T_1}{3\lambda}} + e^{\frac{2\gamma T_1}{3\lambda}} + 2e^{\frac{\gamma T_1}{\lambda}}\Big)\gamma(T_2 - T_1)} X_0.$$

Before investigating the general properties of the two stage model any further, we will review some specific examples. We will return to general statements again in Subsection 4.3.

4.2 Example markets

Our market model is primarily determined by the two parameters γ and λ that were introduced in Section 2. These reflect two different types of costs associated with a large asset transaction:

- A liquidity premium needs to be paid in order to attract buyers *in a short timeframe*. This is quantified by the temporary impact determined by the parameter λ.
- In addition, a premium needs to be paid to attract buyers *at all*. This premium is modeled through the permanent price impact which is characterized by the parameter γ.

In the following, we will analyze two polar market extremes in more detail:

- **Truly illiquid markets**, which are characterized by high transaction costs due to the difficulty to find counterparties quickly (i.e., temporary impact $\lambda >>$ permanent impact γ)
- **Nervous markets**, which are characterized by high transaction costs due to a large sensitivity of the market to transactions (i.e., permanent impact $\gamma >>$ temporary impact λ)

In many practical cases, the market will fall into neither of these two categories, but instead temporary and permanent impact will be balanced; we therefore conclude our case analysis by reviewing a *moderate market*, that is, a market where temporary and permanent impact are balanced: $\lambda \approx \gamma$. For the numerical computations, we used the parameter values given in Table 2.

[3] In the case $n = 1$, it follows from the results in [AC01] and [Alm03] that the optimal trading strategy in stage 2 is a linear increase / decrease of the predator's asset position.

Parameter	Illiquid market	Nervous market	Intermediate market
Asset position X_0		1	
Initial price P_0		10	
Duration T_1 of stage 1		1	
Duration $T_2 - T_1$ of stage 2		1	
Permanent impact sensitivity γ	1	3	1.8
Temporary impact sensitivity λ	3	1	1

Table 2: Parameter values used for numerical computation in Subsection 4.2.

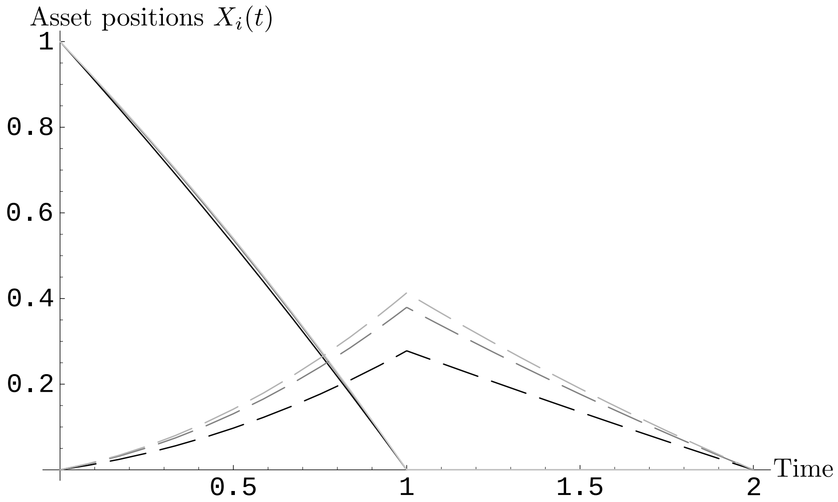

Figure 5: Asset positions $X_i(t)$ over time in a truly illiquid market; at time $t = 1$, stage 1 ends and stage 2 begins. The solid lines represents the seller, the dashed lines the combined asset position of all n predators. The black lines correspond to $n = 2$, the dark grey lines to $n = 10$ and the light grey lines to $n = 100$.

4.2.1 Example market 1: Truly illiquid market

To begin with, let us assume that no predators are active in the market. In such a situation, we would expect that the market price in stage 1 drops dramatically, since in order to satisfy the seller's trading needs, liquidity is required fast - which is expensive in a truly illiquid market. In stage 2, no selling pressure from the seller exists any more; hence, the market price will bounce back. Furthermore, since the permanent impact is comparatively small, it will bounce back almost completely.

A predator knowing of the seller's intentions would expect this price pattern. His natural reaction would therefore be to buy some of the seller's shares in stage 1 at the very low price and to sell them in stage 2 at the much higher price. Figure 5 shows that this is indeed what happens when the seller and the predators follow their optimal strategies.

As can be seen in these figures, the total asset position $\sum_{i=1}^{n} X_{i,1}(T_1)$ acquired by the predators at the end of stage 1 increases as the number of predators increases (see also Figure 6). This can be explained by the following effect: If one predator is acting by himself, he will only buy a comparatively small fraction of the seller's assets, since buying an additional share increases the market price in stage 1 and thus dilutes the profit per share. As the number of predators increases, each predator is buying only a small number of shares and therefore the diluting effect becomes less significant.

The effect of the predators' trading (buying in stage 1, selling in stage 2) is such that the prices between stage 1 and stage 2 will even out; the large price jumps expected in the absence of

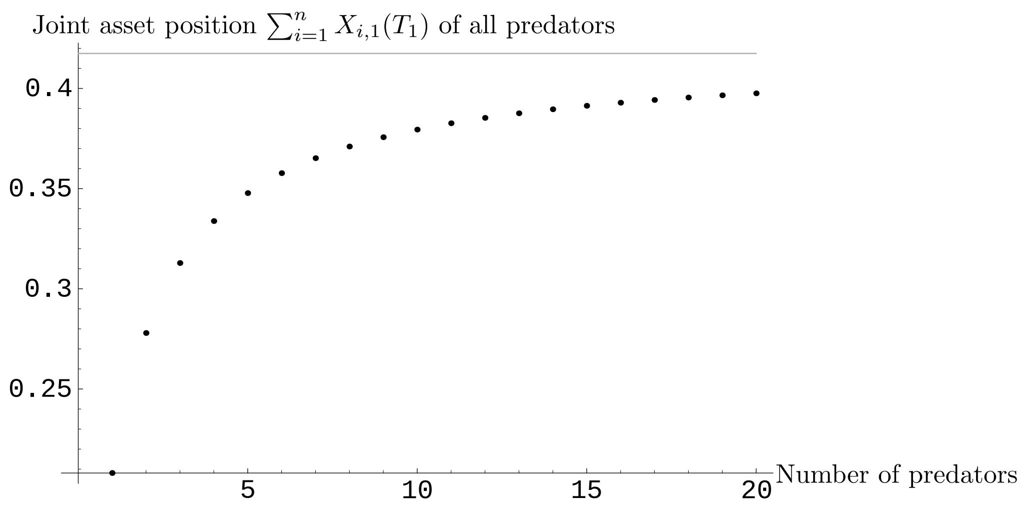

Figure 6: Joint asset position $\sum_{i=1}^{n} X_{i,1}(T_1)$ of all predators in a truly illiquid market at time T_1 depending on the total number n of all predators. The grey line represents the limit $\lim_{n\to\infty} \sum_{i=1}^{n} X_{i,1}(T_1)$.

predators will disappear if the number of predators is large enough (see Figure 7).

From the seller's perspective, the predators' trading is beneficial; by buying some of his shares, the predators reduce the seller's market impact and thus increase his return. As we have just discussed, a larger number of predators implies a larger combined purchase of the predators; hence, the seller can expect to profit from each additional predator, i.e., the larger the number of predators, the larger his profit. This is illustrated by Figure 8; the seller's return is higher when predators are active than it is when there are no predators.

The cooperative behavior between the predators and the seller cannot be explained in the single stage framework described in [CLV05] nor in the market model introduced by [BP05].

The practical implications are evident: in a truly illiquid market, it is sensible to announce any large, time-constrained asset transaction directly at the beginning of trading in order to attract liquidity.

4.2.2 Example market 2: Nervous market

We will now turn to nervous markets, i.e., markets with a permanent impact that clearly exceeds the temporary impact. In such a setting, we expect the price dynamics to be very different from the dynamics described for truly illiquid markets in the previous subsection.

Let us again assume that none of the predators are active. In stage 1, the seller is constantly pushing the market price further and further down; we therefore expect the price to be high at the beginning of stage 1 and low at the end of stage 1. In stage 2, the price will bounce back, since the temporary impact of the seller's trading vanishes. However, this jump will be comparatively small because the temporary price impact is small.

For a predator, this implies that buying some of the seller's shares in stage 1 does not promise any large profit; the price reversion in stage 2 is too small. Instead, it appears more profitable to exploit the price changes *within* stage 1 instead of the price changes *between* stage 1 and stage 2. By selling short the asset at the beginning of stage 1 and buying it back at the end of stage 1, he can likely make a large profit. Thus, we expect to see preying behavior similar to the behavior in the one stage framework discussed in Section 3. Our hypothesis is verified by the numerical results shown in Figure 9.

Similar to the results of Section 3, we might be tempted to expect that the return for the seller is again decreasing as the number of predators increases and predation becomes more fierce. Figure

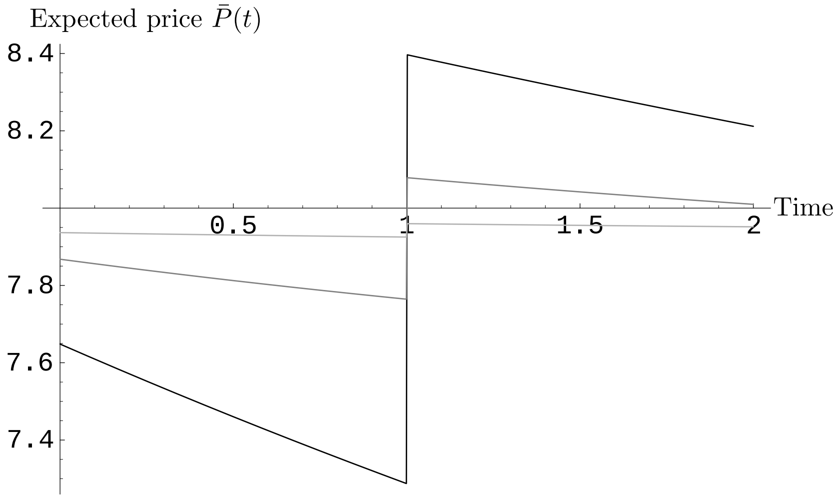

Figure 7: Expected price $\bar{P}(t)$ in a truly illiquid market over time depending on the number of predators n; at time $t = 1$, stage 1 ends and stage 2 begins. The black line corresponds to $n = 2$, the dark grey line to $n = 10$ and the light grey line to $n = 100$. A significant reduction in price drift can be observed; furthermore, $\bar{P}(0)$ is smaller than $P_0 = 10$.

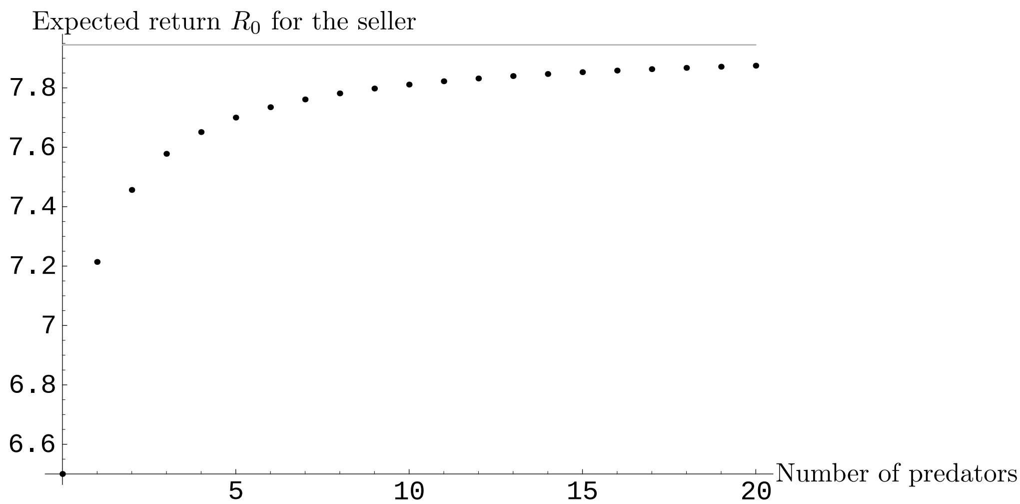

Figure 8: Expected return R_0 for the seller in a truly illiquid market, depending on the number of predators. The grey line represents the limit $n \to \infty$. The return for the seller without predators is at the intersection of x- and y-axis.

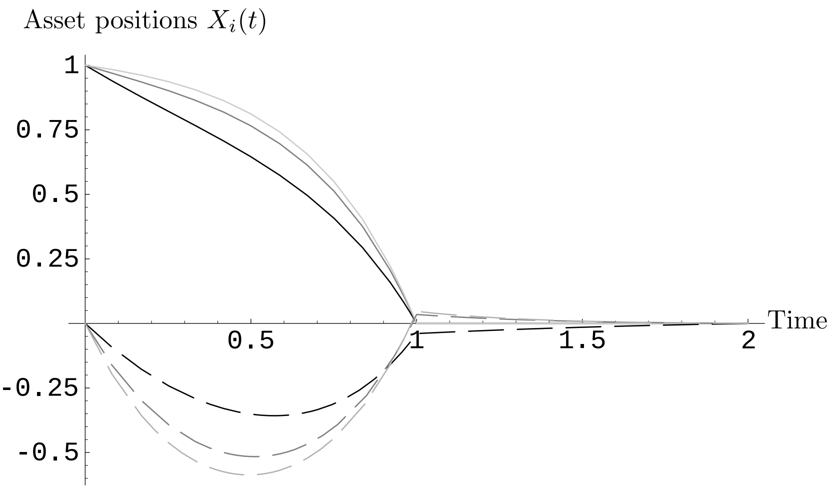

Figure 9: Asset positions $X_i(t)$ over time in a nervous market; at time $t = 1$, stage 1 ends and stage 2 begins. The solid lines represents the seller, the dashed lines the combined asset position of all n predators. The black lines correspond to $n = 2$, the dark grey lines to $n = 10$ and the light grey lines to $n = 100$.

10 shows that this is not the case. The return for the seller is significantly decreased by predators; furthermore, two predators decrease it more than a single predator. However, the return for the seller is higher when three predators are active than when only two predators are active; as soon as at least two predators are active, each additional predator is beneficial for the seller.

The connection between the return for the seller and the number of predators is a combination of effects from the single stage model and the two stage model in a truly illiquid market: If the number of predators is small, the dominant effect is the increasing aggressiveness of predation. As shown in Proposition 2, this results in a flattened price curve within stage 1 (see also Figure 11). In comparison, the recovery of prices between stage 1 and stage 2 now becomes attractive, even though it is relatively small. Similar to the line of argument in truly illiquid markets, it now pays off for the predators to acquire a small asset position during stage 1 in order to sell it during stage 2. This is illustrated in Figure 12. If the number of predators is small, it is beneficial to enter stage 2 with a short position; if the number of predators is large, it is more attractive to enter stage 2 with a long position. Acquiring this long position reduces the seller's price impact in stage 1 and thus increases his return; selling the long position in stage 2 depresses the market price in stage 2 such that the prices in stage 1 and stage 2 are brought in line (see Figure 11).

4.2.3 Example market 3: Intermediate market

In most cases, the differences between the temporary and permanent impact factors γ and λ will not be as extreme as depicted above. If the two parameters are closer together, we can expect to observe characteristics of both truly illiquid as well as nervous markets:

- At the beginning of the first stage, the predators "race the seller to market", that is, they sell in parallel to him. We say that *intra-stage predation* occurs.
- For a large number of predators, the predators will buy back more shares than they sold at the beginning of stage 1; we say that *inter-stage cooperation* takes place in parallel to the intra-stage predation.
- If a certain minimum number of predators is active, additional predators will increase the return for the seller since the increase in inter-stage cooperation outweighs the increase in intra-stage predation.

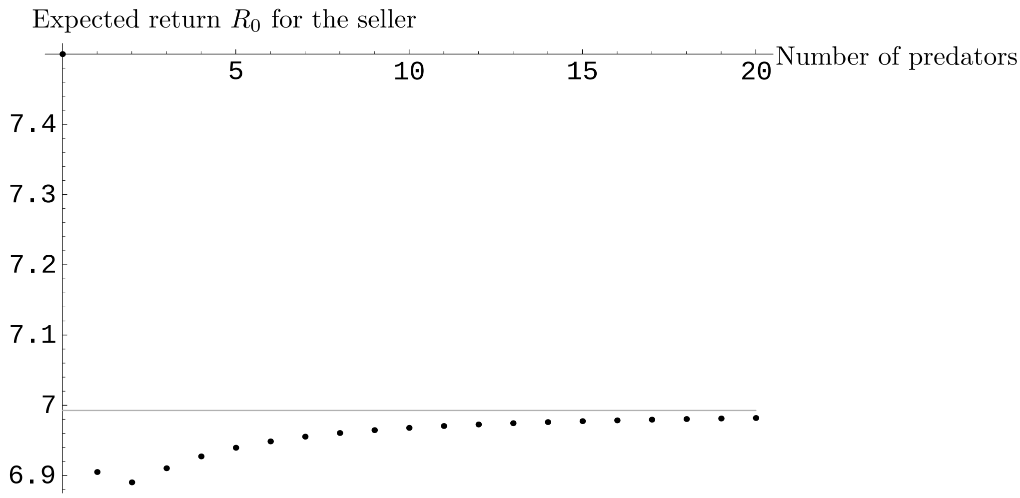

Figure 10: Expected return R_0 for the seller in a nervous market, depending on the number of predators. The grey line represents the limit $n \to \infty$. The return for the seller without predators is at the intersection of x- and y-axis.

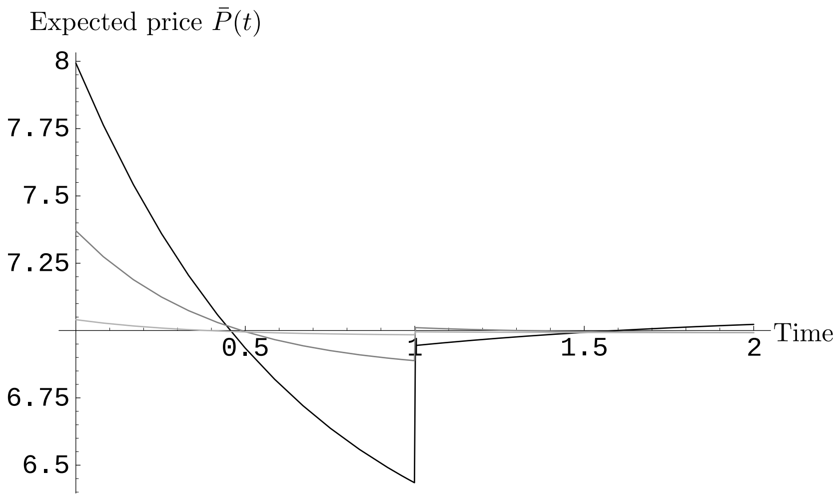

Figure 11: Expected price $\bar{P}(t)$ in a nervous market over time depending on the number of predators n; at time $t = 1$, stage 1 ends and stage 2 begins. The black line corresponds to $n = 2$, the dark grey line to $n = 10$ and the light grey line to $n = 100$. A significant reduction in price drift can be observed.

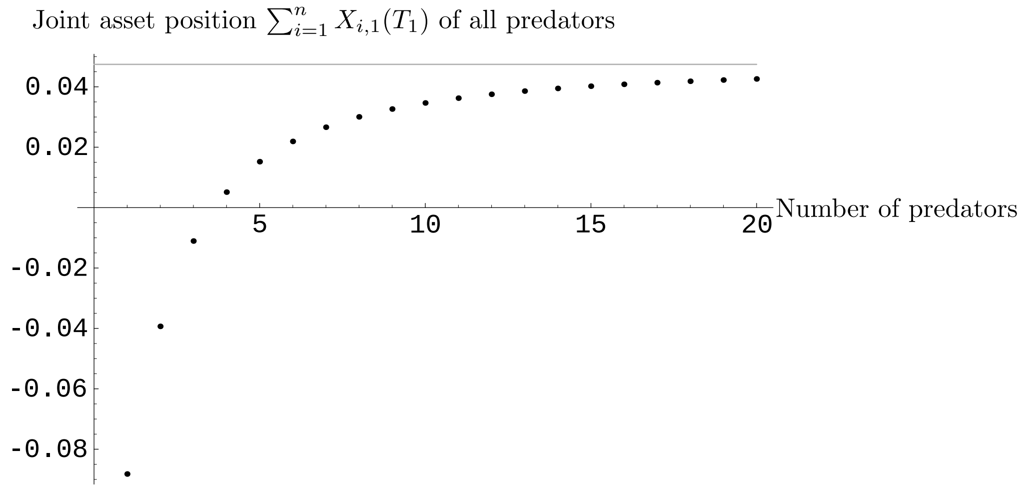

Figure 12: Joint asset position $\sum_{i=1}^{n} X_{i,1}(T_1)$ of all predators in a nervous market at time T_1 depending on the total number n of all predators. The grey line represents the limit $\lim_{n\to\infty} \sum_{i=1}^{n} X_{i,1}(T_1)$.

- If the number of predators is large, market prices will be almost flat and almost the same in stage 1 and stage 2.

All of these characteristics hold; we prove them in general in the next section. However, one interesting question remains open so far. We have already seen that in truly illiquid markets the seller benefits from predators, whereas in nervous markets the seller prefers to have no predators at all. What is the situation in an intermediate market? Of course, both of the phenomena above can be exhibited in an intermediate market, depending on whether it is more nervous or more truly illiquid in nature. However, a combination not considered so far can arise: It might be the case that a small number of predators is harmful to the seller's profits, but a large number is beneficial (see Figure 13 for an example).

The practical implications are evident: If the seller expects to be able to attract a sufficient number of predators, announcing his trading intentions can be attractive; if there is only a limited number of potential predators he is best advised to conceal his intentions.

4.3 General properties

After having reviewed three explicit market examples, let us now summarize their common equilibrium properties regardless of the market parameters.

Proposition 6. *As the number of predators n tends to infinity, the combined asset position of all predators at the end of stage 1 converges to*

$$\lim_{n\to\infty} \sum_{i=1}^{n} X_{i,1}(T_1) = \lim_{n\to\infty} nX_{1,1}(T_1) = \frac{e^{\frac{\gamma(T_2-T_1)}{\lambda}} - 1}{e^{\frac{\gamma(T_2)}{\lambda}} - 1} X_0.$$

In economic terms, this implies that for large n, intra-stage cooperation between the seller and the predators always occurs: in stage 1, the predators buy a portion of the seller's asset position and sell this portion in stage 2. Thereby the market impact in stage 1 is reduced.

We can draw an intuitive consequence for truly illiquid markets: If the number of predators is high, then the net sale of seller and predators in each stage is proportional to the time available for selling. The following corollary expresses this in mathematical terms when sending λ to ∞.

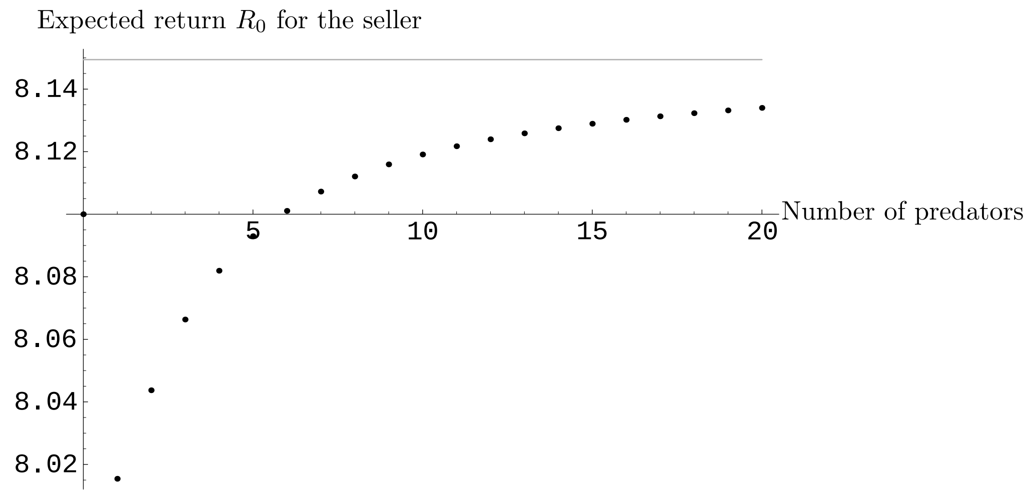

Figure 13: Expected return R_0 for the seller in an intermediate market, depending on the number of predators. The grey line represents the limit $n \to \infty$. The return for the seller without predators is at the intersection of x- and y-axis.

Corollary 7. *As the number of predators n and the temporary price impact coefficient λ tend to infinity, the combined asset position of all predators at time T_1 converges:*

$$\lim_{\lambda\to\infty}\lim_{n\to\infty}\sum_{i=1}^{n} X_{i,1}(T_1) = \frac{T_2 - T_1}{T_2} X_0$$

For the price dynamics we obtain a result similar to Proposition 2: As the number n of predators increases, the price evolution exhibits less drift, i.e., the intentions of the seller are incorporated into market prices more quickly.

Proposition 8. *The drift $|\dot{\bar{P}}_1(t)|$ (respectively $|\dot{\bar{P}}_2(t)|$) is a decreasing function of n. In the limit, the expected market price instantaneously jumps to*

$$P_0 - \frac{\gamma}{1 - e^{-\frac{\gamma(T_2)}{\lambda}}} X_0$$

and is constant from there on throughout stage 1 and stage 2 until the end of stage 2.

It is interesting to note that the new equilibrium price $P_0 - \frac{\gamma}{1-e^{-\frac{\gamma(T_2)}{\lambda}}} X_0$ does not depend on whether the seller can trade in stage 2 (see Proposition 2). In nervous markets, the initial price jump $|\bar{P}_1(0) - P_0|$ is an increasing function of the number n of predators, similar to the behavior in the one stage model. On the other hand, it is a decreasing function of n in truly illiquid markets.

Theorem 9. *By selling his asset position X_0 in stage 1, the seller receives an average total cash position of*

$$R_0 = X_0 \left(P_0 - \gamma X_0 \frac{A_7 n^7 + A_6 n^6 + A_5 n^5 + A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0}{B_7 n^7 + B_6 n^6 + B_5 n^5 + B_4 n^4 + B_3 n^3 + B_2 n^2 + B_1 n + B_0} \right).$$

The coefficients A_i and B_i are functions of n that converge in the limit $n \to \infty$ (see the Appendix for explicit formulas). For large n, the seller's return is increasing in n and converges to:

$$\lim_{n\to\infty} R_0 = X_0 \left(P_0 - \gamma X_0 \frac{1}{1 - e^{-\frac{\gamma}{\lambda}(T_2)}} \right)$$

Given the result above, the benefits of sunshine trading can easily be quantified. If the seller's intentions remain secret, he can expect a return of[4]

$$X_0 \left(P_0 - \gamma X_0/2 - \lambda X_0/T_1\right).$$

Alternatively, he can pre-announce his intentions, attract a large number of predators and thus expect a return of

$$X_0 \left(P_0 - \gamma X_0 \frac{1}{1 - e^{-\frac{\gamma}{\lambda}(T_2)}}\right).$$

We can conclude that pre-announcing is beneficial if

$$\frac{1}{2} + \frac{\lambda}{\gamma T_1} > \frac{1}{1 - e^{-\frac{\gamma}{\lambda}(T_2)}}.$$

If the predators do not face any material time constraint, we can consider the limit case $T_2 \to \infty$ and obtain that pre-announcement is beneficial if

$$\frac{\lambda}{\gamma} > \frac{T_1}{2}.$$

In our model, only the ratio γ/λ of the market parameters γ and λ and the length of the two stages T_1 and $T_2 - T_1$ determine whether sunshine trading is beneficial. This result is model dependent; in the model used by Admati and Pfleiderer [AP91], sunshine trading is always beneficial, while in the model of Brunnermeier and Pedersen [BP05], the benefits of sunshine trading depend on the size of the order but not on the market characteristics.

5 Summary and Conclusions

In a number of practical cases, investors need to liquidate large asset positions in a short time. In this paper, we describe optimal liquidation strategies in case other market participants are aware of the investor's needs. A crucial assumption is that these predators are not limited by the same time constraint that the seller is facing.

We solve a competitive trading game in an illiquid market model incorporating a temporary and a permanent price impact. Each player faces a continuous-time, dynamic programming problem. According to our model, the optimal strategies for these predators depend on the illiquidity characteristics of the market. If the permanent impact affects market prices more heavily than the permanent impact does, the predators will "race" the seller to market, selling in parallel with him and buying back after the seller sold his asset position. If the temporary impact is the dominant illiquidity effect, predators will provide liquidity to the seller by buying some of his shares and selling them after the seller has finished his sale. In the latter case, pre-announcing a trade can be beneficial in order to attract liquidity suppliers.

We then investigate the market behavior in case a very large number of predators is active. We find that in spite of illiquidity, the market efficiently determines a new price. Information about the seller's intentions is immediately incorporated into the market price and does not affect it thereafter. The predators might race the seller to market, but even in markets with high permanent impact, they quickly start buying back shares and sell these after the seller has finished his sale.

In conclusion, we believe that our model enhances the understanding of liquidity provision and predation in the marketplace. It provides a method to decide whether large trades should be pre-announced or kept secret.

[4] See [AC01] and [Alm03] for a discussion of the case without predators.

A Derivations

Proof of Proposition 2. Using the notation from Theorem 1, the combined trading speed of the seller and all predators amounts to

$$\sum_{i=0}^{n} \dot{X}_i(t) = \sum_{i=0}^{n} (a e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t} + b_i e^{\frac{\gamma}{\lambda}t}) = (n+1) a e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t}.$$

The change in combined asset position at time t is therefore:

$$\begin{aligned}
\sum_{i=0}^{n}(X_i(t) - X_i(0)) &= \sum_{i=0}^{n}\int_0^t \dot{X}_i(s)ds \\
&= \int_0^t \sum_{i=0}^{n} \dot{X}_i(s)ds \\
&= \int_0^t (n+1) a e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}s} ds \\
&= (n+1)\frac{n+2}{n}\frac{\lambda}{\gamma} a (1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t})
\end{aligned}$$

Now, we can compute the expected market price:

$$\begin{aligned}
\bar{P}(t) &= P_0 + \gamma \sum_{i=0}^{n}(X_i(t) - X_i(0)) + \lambda \sum_{i=0}^{n} \dot{X}_i(t) \\
&= P_0 + \gamma(n+1)\frac{n+2}{n}\frac{\lambda}{\gamma} a(1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t}) + \lambda(n+1) a e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t} \\
&= P_0 + \lambda\frac{n+1}{n}(n+2-2e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t})a \\
&= P_0 + \lambda\frac{n+1}{n}(n+2-2e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t})\frac{n}{n+2}\frac{\gamma}{\lambda}\left(1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}\right)^{-1}\frac{-X_0}{n+1} \\
&= P_0 - \gamma X_0 \frac{1}{1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}} + \gamma X_0 \frac{2}{n+2}\frac{e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t}}{1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}}
\end{aligned}$$

Only the last term in the expression above is time dependent; its influence decreases with increasing n. In the limit, we obtain that the expected market price $\bar{P}(t)$ is constant:

$$\lim_{n\to\infty} \bar{P}(t) \equiv P_0 - \gamma X_0 \frac{1}{1 - e^{-\frac{\gamma}{\lambda}T_1}}$$

The initial price jump amounts to

$$P_0 - \bar{P}(0) = \gamma X_0 \frac{n}{n+2}\frac{1}{1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}}$$

which is easily seen to be increasing in n. □

Proof of Proposition 3. Without any predators, the optimal strategy for the seller is to liquidate his asset position linearly: $X_0(t) = (T_1 - t)X_0/T_1$ (for the easy proof, see [AC01] or [Alm03]). The market price thus drops to

$$\bar{P}(T_1) = P_0 - \gamma X_0 - \lambda X_0/T_1.$$

Price overshooting thus amounts to $\lambda X_0/T_1$.

From the proof of Proposition 2, we know the structure of $\bar{P}(t)$ when predators are present and deduce that the market price monotonously falls to

$$\bar{P}(T_1) = P_0 - \gamma X_0 \frac{1}{1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}} + \gamma X_0 \frac{2}{n+2} \frac{e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}}{1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}}.$$

Thus, the price overshoots with magnitude

$$\bar{P}(T_1+) - \bar{P}(T_1) = \frac{n}{n+2}\gamma X_0 \frac{e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}}{1 - e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}}.$$

The monotonicity follows directly. □

Proof of Proposition 4. Using Theorem 1 and Proposition 2 and applying the notation introduced in Proposition 4, we can calculate in a straightforward way:

$$\begin{aligned}
R_0 &= \int_0^{T_1} (-\dot{X}_0(t))\bar{P}(t)dt \\
&= -\int_0^{T_1} \dot{X}_0(t)\left(P_0 - \gamma X_0 \frac{1}{1-1/p} + \gamma X_0 \frac{2}{n+2}\frac{e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t}}{1-1/p}\right)dt \\
&= X_0\left(P_0 - \gamma X_0 \frac{1}{1-1/p}\right) \\
&\quad -\gamma X_0 \frac{2}{(n+2)(1-1/p)} \int_0^{T_1} (ae^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t} + b_0 e^{\frac{\gamma}{\lambda}t})e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t}dt \\
&= X_0\left(P_0 - \gamma X_0 \frac{1}{1-1/p}\right) \\
&\quad -\gamma X_0 \frac{2}{(n+2)(1-1/p)} \left(a\frac{n+2}{2n}\frac{\lambda}{\gamma}(1-e^{-\frac{2n}{n+2}\frac{\gamma}{\lambda}T_1}) + b_0 \frac{n+2}{2}\frac{\lambda}{\gamma}(-1+e^{\frac{2}{n+2}\frac{\gamma}{\lambda}T_1})\right) \\
&= X_0\left(P_0 - \gamma X_0 \frac{1}{1-1/p}\right) - \lambda X_0 \frac{1}{1-1/p}\left(a\frac{1}{n}(1-1/p^2) + b_0(-1+q/p)\right) \\
&= X_0\left(P_0 - \gamma X_0 \frac{1}{1-1/p}\right) - \lambda X_0 \frac{1}{1-1/p} \\
&\quad \left(\frac{n}{n+2}\frac{\gamma}{\lambda}(1-1/p)^{-1}\frac{-X_0}{n+1}\frac{1}{n}(1-1/p^2) + \frac{\gamma}{\lambda}(q-1)^{-1}(-X_0\frac{n}{n+1})(-1+q/p)\right) \\
&= X_0\left(P_0 - \gamma X_0 \frac{1}{1-1/p}\right) + \gamma X_0^2 \frac{1}{1-1/p}\left(\frac{1+1/p}{(n+2)(n+1)} + \frac{n(-1+q/p)}{(n+1)(q-1)}\right)
\end{aligned}$$

Extending the last expression to the common denominator and grouping by powers of n produces the desired result.

For the limit $n \to \infty$, we calculate

$$\begin{aligned}
\lim_{n\to\infty} R_0 &= \lim_{n\to\infty} X_0\left(P_0 - \gamma X_0 \frac{A_2 n^2 + A_1 n + A_0}{B_2 n^2 + B_1 n + B_0}\right) \\
&= X_0\left(P_0 - \gamma X_0 \lim_{n\to\infty}\frac{A_2}{B_2}\right) \\
&= X_0\left(P_0 - \gamma X_0 \frac{q}{q-1}\right).
\end{aligned}$$

In the second line above, we have used the fact that A_0, A_1, B_0 and B_1 converge in the limit $n \to \infty$.

□

Proof of Theorem 5. The actual computations are lengthy; we will therefore only sketch the approach.

Let us first discuss the case $n = 1$, i.e., the seller is facing only one predator. By computations similar to Proposition 2, we can express the expected market price $\bar{P}(t)$ as a *linear* function of the seller's asset position X_0 and the predators asset position $Z_1 = X_{1,1}(T_1)$ at the end of stage 1. Similar to Proposition 4, we can then calculate the return for the predator in the two stages as a *quadratic* function of X_0 and Z_1:

$$Return_{Predator} = Return_{Stage1}(X_0, Z_1) + Return_{Stage2}(X_0, Z_1)$$

Now, we can determine the optimal Z_1 by maximizing the quadratic function $Return_{Predator}$, i.e., by determining the root of its derivative, which is a linear function in X_0.

Let us now turn to the case $n \geq 2$, i.e., the seller is facing at least two predators. By symmetry, the optimal strategies for all predators are identical; therefore, we know that $X_{i,1}(T_1) = X_{j,1}(T_1) =: Y_1$ for all $i, j \neq 0$. In order to determine this optimal value Y_1, we assume that $n-1$ predators 1, 2, ..., $n-1$ are acquiring the optimal asset position Y_1 during stage 1, while the last predator $i = n$ is acquiring an asset position Z_1. Similar to the case $n = 1$ discussed above, we can calculate the return for the last predator as a *quadratic* function of X_0, Y_1 and Z_1:

$$Return_{Predator_n} = Return_{Stage1}(X_0, Y_1, Z_1) + Return_{Stage2}(X_0, Y_1, Z_1)$$

We can again determine the optimal Z_1 by maximizing $Return_{Predator_n}$ and obtain a *linear* function of X_0 and Y_1:

$$Z_1^{optimal} = f(X_0, Y_1)$$

Since we assumed that Y_1 was optimal in the first place, we know that the optimal $Z_1^{optimal}$ has to be equal to Y_1; solving $Y_1 = f(X_0, Y_1)$ for Y_1 gives the value of Y_1 as a *linear* function of X_0. We obtain that

$$\begin{aligned} A_0 \;=\; & -2\Bigg(e^{\frac{\gamma((1+n)T_1+(T_2-T_1))}{(1+n)\lambda}} - e^{\frac{\gamma((1+n)T_1+n(T_2-T_1))}{(1+n)\lambda}} - e^{\frac{\gamma((1+n)T_1+(2+n)(T_2-T_1))}{(1+n)\lambda}} - \\ & e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)(T_2-T_1)\right)}{\left(2+3n+n^2\right)\lambda}} + e^{\frac{\gamma\left(2(1+n)^2T_1+n(2+n)(T_2-T_1)\right)}{\left(2+3n+n^2\right)\lambda}} + \\ & e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)^2(T_2-T_1)\right)}{\left(2+3n+n^2\right)\lambda}} + e^{\frac{\gamma((1+n)T_1+(1+2n)(T_2-T_1))}{(1+n)\lambda}} - \\ & e^{\frac{\gamma\left(2(1+n)^2T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{\left(2+3n+n^2\right)\lambda}} \Bigg) \end{aligned}$$

$$\begin{aligned} A_1 \;=\; & -3e^{\frac{(2+n)\gamma(T_2-T_1)}{(1+n)\lambda}} + 3e^{\frac{(1+2n)\gamma(T_2-T_1)}{(1+n)\lambda}} + 3e^{\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} - 3e^{\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} - \\ & 2e^{\frac{\gamma((1+n)T_1+(T_2-T_1))}{(1+n)\lambda}} + 2e^{\frac{\gamma((1+n)T_1+n(T_2-T_1))}{(1+n)\lambda}} + 2e^{\frac{\gamma((1+n)T_1+(2+n)(T_2-T_1))}{(1+n)\lambda}} + \\ & e^{\frac{n\gamma((1+n)T_1+(2+n)(T_2-T_1))}{\left(2+3n+n^2\right)\lambda}} - e^{\frac{\gamma(n(1+n)T_1+(2+n)(T_2-T_1))}{\left(2+3n+n^2\right)\lambda}} + \\ & e^{\frac{\gamma\left(n(1+n)T_1+(2+n)^2(T_2-T_1)\right)}{\left(2+3n+n^2\right)\lambda}} - 2e^{\frac{\gamma((1+n)T_1+(1+2n)(T_2-T_1))}{(1+n)\lambda}} - \\ & e^{\frac{\gamma\left(n(1+n)T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{\left(2+3n+n^2\right)\lambda}} \end{aligned}$$

$$A_2 = -e^{\frac{(2+n)\gamma(T_2-T_1)}{(1+n)\lambda}} + e^{\frac{(1+2n)\gamma(T_2-T_1)}{(1+n)\lambda}} + e^{\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} - e^{\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{n\gamma((1+n)T_1+(2+n)(T_2-T_1))}{\left(2+3n+n^2\right)\lambda}} - e^{\frac{\gamma(n(1+n)T_1+(2+n)(T_2-T_1))}{\left(2+3n+n^2\right)\lambda}} + e^{\frac{\gamma\left(n(1+n)T_1+(2+n)^2(T_2-T_1)\right)}{\left(2+3n+n^2\right)\lambda}} - e^{\frac{\gamma\left(n(1+n)T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{\left(2+3n+n^2\right)\lambda}}$$

and

$$B_0 = -2\left(2e^{\frac{(1+2n)\gamma(T_2-T_1)}{(1+n)\lambda}} - e^{\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} - e^{\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{\gamma((1+n)T_1+n(T_2-T_1))}{(1+n)\lambda}} - e^{\frac{\gamma((1+n)T_1+(2+n)(T_2-T_1))}{(1+n)\lambda}} + e^{\frac{n\gamma((1+n)T_1+(2+n)(T_2-T_1))}{(1+n)(2+n)\lambda}} - e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} - e^{\frac{\gamma\left(2(1+n)^2T_1+n(2+n)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)^2(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} - e^{\frac{\gamma((1+n)T_1+(1+2n)(T_2-T_1))}{(1+n)\lambda}} - 2e^{\frac{\gamma\left(n(1+n)T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + e^{\frac{\gamma\left(2(1+n)^2T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + 2e^{\frac{\gamma T_1}{\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} - e^{\frac{2(1+n)\gamma T_1}{(2+n)\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{n\gamma T_1}{2\lambda+n\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} - e^{\frac{\gamma T_1}{\lambda}+\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{2(1+n)\gamma T_1}{(2+n)\lambda}+\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}}\right)$$

$$B_1 = 2e^{\frac{(2+n)\gamma(T_2-T_1)}{(1+n)\lambda}} - e^{\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} - e^{\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} - e^{\frac{\gamma((1+n)T_1+n(T_2-T_1))}{(1+n)\lambda}} - e^{\frac{\gamma((1+n)T_1+(2+n)(T_2-T_1))}{(1+n)\lambda}} + e^{\frac{n\gamma((1+n)T_1+(2+n)(T_2-T_1))}{(1+n)(2+n)\lambda}} - 3e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + e^{\frac{\gamma\left(2(1+n)^2T_1+n(2+n)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} - 2e^{\frac{\gamma\left(n(1+n)T_1+(2+n)^2(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)^2(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} - e^{\frac{\gamma((1+n)T_1+(1+2n)(T_2-T_1))}{(1+n)\lambda}} + e^{\frac{\gamma\left(2(1+n)^2T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + 3e^{\frac{2(1+n)\gamma T_1}{(2+n)\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{n\gamma T_1}{2\lambda+n\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + 3e^{\frac{\gamma T_1}{\lambda}+\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} - 3e^{\frac{2(1+n)\gamma T_1}{(2+n)\lambda}+\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}}$$

$$B_2 = 2\left(e^{\frac{(2+n)\gamma(T_2-T_1)}{(1+n)\lambda}} - 2e^{\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} - e^{\frac{\gamma((1+n)T_1+n(T_2-T_1))}{(1+n)\lambda}} - 2e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + e^{\frac{\gamma\left(2(1+n)^2T_1+n(2+n)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} - e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)^2(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} - e^{\frac{\gamma((1+n)T_1+(1+2n)(T_2-T_1))}{(1+n)\lambda}} - e^{\frac{\gamma\left(n(1+n)T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + 2e^{\frac{\gamma\left(2(1+n)^2T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + e^{\frac{\gamma T_1}{\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + 2e^{\frac{2(1+n)\gamma T_1}{(2+n)\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{n\gamma T_1}{2\lambda+n\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{\gamma T_1}{\lambda}+\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} - 2e^{\frac{2(1+n)\gamma T_1}{(2+n)\lambda}+\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}}\right)$$

$$
\begin{aligned}
B_3 \;=\;& -e^{\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} - e^{\frac{\gamma((1+n)T_1+n(T_2-T_1))}{(1+n)\lambda}} + e^{\frac{\gamma((1+n)T_1+(2+n)(T_2-T_1))}{(1+n)\lambda}} - \\
& e^{\frac{n\gamma((1+n)T_1+(2+n)(T_2-T_1))}{(1+n)(2+n)\lambda}} - e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + \\
& e^{\frac{\gamma\left(2(1+n)^2T_1+n(2+n)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} - e^{\frac{\gamma\left(2(1+n)^2T_1+(2+n)^2(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} - \\
& e^{\frac{\gamma((1+n)T_1+(1+2n)(T_2-T_1))}{(1+n)\lambda}} + e^{\frac{\gamma\left(2(1+n)^2T_1+\left(2+5n+2n^2\right)(T_2-T_1)\right)}{(1+n)(2+n)\lambda}} + \\
& e^{\frac{2(1+n)\gamma T_1}{(2+n)\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{n\gamma T_1}{2\lambda+n\lambda}+\frac{\gamma(T_2-T_1)}{\lambda+n\lambda}} + e^{\frac{\gamma T_1}{\lambda}+\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}} - \\
& e^{\frac{2(1+n)\gamma T_1}{(2+n)\lambda}+\frac{n\gamma(T_2-T_1)}{\lambda+n\lambda}}.
\end{aligned}
$$

The convergence of $\lim_{n\to\infty} A_i$ and $\lim_{n\to\infty} B_i$ can be established by direct calculations. We obtain:

$$
\begin{aligned}
\lim_{n\to\infty} A_0 &= 2e^{\frac{\gamma T_1}{\lambda}}\Big(-1+e^{\frac{\gamma T_1}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma(T_2-T_1)}{\lambda}}\Big)^2 \\
\lim_{n\to\infty} A_1 &= -3\Big(-1+e^{\frac{\gamma T_1}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma(T_2-T_1)}{\lambda}}\Big)^2 \\
\lim_{n\to\infty} A_2 &= -\Big(-1+e^{\frac{\gamma T_1}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma(T_2-T_1)}{\lambda}}\Big)^2 \\
\lim_{n\to\infty} B_0 &= -2\Big(-1+e^{\frac{\gamma T_1}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma(T_2-T_1)}{\lambda}}\Big)\Big(-1+2e^{\frac{\gamma T_1}{\lambda}}-2e^{\frac{\gamma(T_2-T_1)}{\lambda}}+e^{\frac{\gamma T_2}{\lambda}}\Big) \\
\lim_{n\to\infty} B_1 &= \Big(-1+e^{\frac{\gamma T_1}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma(T_2-T_1)}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma T_2}{\lambda}}\Big) \\
\lim_{n\to\infty} B_2 &= 4\Big(-1+e^{\frac{\gamma T_1}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma(T_2-T_1)}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma T_2}{\lambda}}\Big) \\
\lim_{n\to\infty} B_3 &= \Big(-1+e^{\frac{\gamma T_1}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma(T_2-T_1)}{\lambda}}\Big)\Big(-1+e^{\frac{\gamma T_2}{\lambda}}\Big)
\end{aligned}
$$

Note that the denominator $B_3n^3 + B_2n^2 + B_1n + B_0$ of the general expression

$$X_{i,1}(T_1) = -\frac{A_2n^2 + A_1n + A_0}{B_3n^3 + B_2n^2 + B_1n + B_0} X_0$$

is 0 in the case $n = 1$; however, the general expression as a whole converges for $n \to 1$ against the optimal value of $X_{1,1}(T_1)$ for $n = 1$. □

Proof of Proposition 6. We apply Theorem 5 and obtain:

$$\lim_{n\to\infty} \sum_{i=1}^{n} X_{i,1}(T_1) = -\frac{\lim_{n\to\infty} A_2}{\lim_{n\to\infty} B_3} X_0$$

From the proof of Theorem 5, we know the values of the limits of A_2 and B_3 and the desired result follows. □

Proof of Corollary 7. Using Proposition 6 and L'Hospitale's rule, we calculate

$$\lim_{\lambda\to\infty} \lim_{n\to\infty} \sum_{i=1}^{n} X_{i,1}(T_1) = \lim_{\lambda\to\infty} \frac{e^{\frac{\gamma(T_2-T_1)}{\lambda}} - 1}{e^{\frac{\gamma T_2}{\lambda}} - 1} = \frac{T_2 - T_1}{T_2}$$

□

Proof of Proposition 8. First, we note that by arguments similar to the proof of Proposition 2, the price during stage 1 is

$$
\begin{aligned}
\bar{P}_1(t) &= P_0 - \gamma\left(X_0 - \sum_{i=1}^{n} X_{i,1}(T_1)\right)\frac{1}{1-e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}} \\
&\quad +\gamma\left(X_0 - \sum_{i=1}^{n} X_{i,1}(T_1)\right)\frac{2}{n+2}\frac{e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t}}{1-e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}}
\end{aligned}
$$

and the price during stage 2 is

$$
\begin{aligned}
\bar{P}_2(t) &= P_0 - \gamma\left(X_0 - \sum_{i=1}^{n} X_{i,1}(T_1)\right) - \gamma\left(\sum_{i=1}^{n} X_{i,1}(T_1)\right)\frac{1}{1-e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}(T_2-T_1)}} \\
&\quad +\gamma\left(\sum_{i=1}^{n} X_{i,1}(T_1)\right)\frac{2}{n+2}\frac{e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}t}}{1-e^{-\frac{n}{n+2}\frac{\gamma}{\lambda}(T_2-T_1)}}.
\end{aligned}
$$

Again, the time-dependent terms vanish as n increases, and we obtain the limits

$$
\begin{aligned}
\lim_{n\to\infty}\bar{P}_1(t) &= P_0 - \gamma\left(X_0 - \lim_{n\to\infty}\sum_{i=1}^{n} X_{i,1}(T_1)\right)\frac{1}{1-e^{-\lim_{n\to\infty}\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}} \\
&\quad +\gamma\left(X_0 - \lim_{n\to\infty}\sum_{i=1}^{n} X_{i,1}(T_1)\right)\left(\lim_{n\to\infty}\frac{2}{n+2}\right)\frac{e^{-\lim_{n\to\infty}\frac{n}{n+2}\frac{\gamma}{\lambda}t}}{1-e^{-\lim_{n\to\infty}\frac{n}{n+2}\frac{\gamma}{\lambda}T_1}} \\
&= P_0 - \gamma\left(X_0 - \frac{e^{\frac{\gamma(T_2-T_1)}{\lambda}}-1}{e^{\frac{\gamma T_2}{\lambda}}-1}X_0\right)\frac{1}{1-e^{-\frac{\gamma}{\lambda}T_1}} \\
&= P_0 - \gamma X_0\frac{e^{\frac{\gamma T_2}{\lambda}}}{e^{\frac{\gamma T_2}{\lambda}}-1}
\end{aligned}
$$

$$
\begin{aligned}
\lim_{n\to\infty}\bar{P}_2(t) &= P_0 - \gamma\left(X_0 - \lim_{n\to\infty}\sum_{i=1}^{n} X_{i,1}(T_1)\right) \\
&\quad -\gamma\left(\lim_{n\to\infty}\sum_{i=1}^{n} X_{i,1}(T_1)\right)\frac{1}{1-e^{-\lim_{n\to\infty}\frac{n}{n+2}\frac{\gamma}{\lambda}(T_2-T_1)}} \\
&\quad +\gamma\left(\lim_{n\to\infty}\sum_{i=1}^{n} X_{i,1}(T_1)\right)\lim_{n\to\infty}\frac{2}{n+2}\frac{e^{-\lim_{n\to\infty}\frac{n}{n+2}\frac{\gamma}{\lambda}t}}{1-e^{-\lim_{n\to\infty}\frac{n}{n+2}\frac{\gamma}{\lambda}(T_2-T_1)}} \\
&= P_0 - \gamma\left(X_0 - \frac{e^{\frac{\gamma(T_2-T_1)}{\lambda}}-1}{e^{\frac{\gamma T_2}{\lambda}}-1}X_0\right) \\
&\quad -\gamma\left(\frac{e^{\frac{\gamma(T_2-T_1)}{\lambda}}-1}{e^{\frac{\gamma T_2}{\lambda}}-1}X_0\right)\frac{1}{1-e^{-\frac{\gamma}{\lambda}(T_2-T_1)}} \\
&= P_0 - \gamma X_0\frac{e^{\frac{\gamma T_2}{\lambda}}}{e^{\frac{\gamma T_2}{\lambda}}-1}.
\end{aligned}
$$

□

Proof of Theorem 9. Identically to the approach in Proposition 4, the formula for the return for the seller can be derived. The coefficients A_i and B_i are of a similar structure to the ones in Proposition 4, but even more convoluted; we omit them here for brevity.

The coefficients A_i and B_i converge for $n \to \infty$; furthermore, their derivatives $\frac{dA_i}{dn}$ and $\frac{dB_i}{dn}$ converge to 0 as $n \to \infty$. We compute

$$\lim_{n\to\infty} R_0 = \lim_{n\to\infty} \mathbb{E}(\text{Return for the seller}) = X_0 \left(P_0 - \gamma X_0 \frac{\lim_{n\to\infty} A_7}{\lim_{n\to\infty} B_7} \right).$$

Inserting A_7 and B_7 and computing the limit gives the desired limit.

To prove that $\lim_{n\to\infty} R_0$ is increasing for large n, we compute the derivative of the seller's return R_0 with respect to n as

$$\frac{d}{dn} R_0 = -\gamma X_0 \frac{\text{Numerator}}{(B_7 n^7 + B_6 n^6 + B_5 n^5 + B_4 n^4 + B_3 n^3 + B_2 n^2 + B_1 n + B_0)^2}$$

with

$$\begin{aligned} \text{Numerator} \quad = \quad & \Bigg(7A_7 B_7 n + 7A_7 B_6 + 6A_6 B_7 + \frac{dA_7}{dn} B_7 n^2 + \frac{dA_7}{dn} B_6 n \\ & + \frac{dA_7}{dn} B_5 + \frac{dA_6}{dn} B_7 n + \frac{dA_6}{dn} B_6 + \frac{dA_5}{dn} B_7 \Bigg) n^{12} \\ & - \Bigg(7B_7 A_7 n + 7B_7 A_6 + 6B_6 A_7 + \frac{dB_7}{dn} A_7 n^2 + \frac{dB_7}{dn} A_6 n \\ & + \frac{dB_7}{dn} A_5 + \frac{dB_6}{dn} A_7 n + \frac{dB_6}{dn} A_6 + \frac{dB_5}{dn} A_7 \Bigg) n^{12} + o(n^{11}). \end{aligned}$$

For large n, we can omit the $o(n^{11})$ term; furthermore, we know that all derivatives converge to 0 as $n \to \infty$. We therefore obtain for large n:

$$\begin{aligned} \text{Numerator} \quad \approx \quad & \Bigg(\left(\frac{dA_7}{dn} B_7 - \frac{dB_7}{dn} A_7 \right) n^2 \\ & + \left(\frac{dA_7}{dn} B_6 + \frac{dA_6}{dn} B_7 - \frac{dB_7}{dn} A_6 n - \frac{dB_6}{dn} A_7 \right) n \\ & + A_7 B_6 - B_7 A_6 \Bigg) n^{12} \end{aligned}$$

Inserting the expressions for A_i and B_i, we obtain

$$\lim_{n\to\infty} \left(\frac{dA_7}{dn} B_7 - \frac{dB_7}{dn} A_7 \right) n^2 = 0$$

$$\lim_{n\to\infty} \left(\frac{dA_7}{dn} B_6 + \frac{dA_6}{dn} B_7 - \frac{dB_7}{dn} A_6 n - \frac{dB_6}{dn} A_7 \right) n = 0$$

$$\lim_{n\to\infty} (A_7 B_6 - B_7 A_6) = -e^{\frac{\gamma T_1}{\lambda}} \left(e^{\frac{\gamma T_1}{\lambda}} - 1 \right)^7 \left(e^{\frac{\gamma (T_2 - T_1)}{\lambda}} - 1 \right)^5 \left(e^{\frac{\gamma T_2}{\lambda}} - 1 \right)^3 \gamma^2 < 0.$$

The derivative of the seller's return has the opposite sign of the *Numerator* and is thus positive for large values of n. □

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