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Entropy approximation and large deviations

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Let (Ω,τ) be a dynamical system where τ is a $\mathbb{Z}^d$ (resp. $\mathbb{Z}^d_+$) action and Ω the (compact metric) phase space, let f be an element of the set $C(\Omega)$ of real-valued continuous functions on Ω, let (ν_α) be a net of Borel probability measures on the space $\mathcal{M}(\Omega)$ of Borel probability measures on Ω, and let $t_\alpha \downarrow 0$. Assume that any invariant measure can be approximated weakly and in entropy by a net of measures, each one being the unique equilibrium state for some potential. We show that (ν_α) satisfies a strong form of the large deviation principle with powers (t_α) and rate function

$$I^f(\mu) = \begin{cases} P^\tau(f) - \mu(f) - h^\tau_\mu & \text{if } \mu \in \mathcal{M}^\tau(\Omega) \\ \\ +\infty & \text{if } \mu \in \mathcal{M}(\Omega)\backslash\mathcal{M}^\tau(\Omega) \end{cases}$$

(where $P^\tau(\cdot)$, $h^\tau_\cdot$, $\mathcal{M}^\tau(\Omega)$ denote respectively the pressure map, the entropy map, and the set of invariant elements of $\mathcal{M}(\Omega)$) if and only if

$$\lim t_\alpha \log \int_{\mathcal{M}(\Omega)} e^{\mu(g)/t_\alpha} \nu_\alpha(d\mu) = P^\tau(f+g) - P^\tau(f) \quad \text{for all } g \in C(\Omega). \quad (1)$$

Without any assumption, the upper-bounds follow from the mere inequality "$\leq$" in (1) with moreover only a upper-limit in the L.H.S. The hypothesis is known to hold for the iteration of hyperbolic rational maps, and for the multidimensional full shift. In both cases, the above result is applied to various kinds of nets (ν_α). This allows us to strengthen several known results, and in particular those of statistical mechanics concerning Gibbs fields.