

INTERNATIONAL CONFERENCE ON DYNAMICAL SYSTEMS  
IN HONOUR OF MICHAŁ MISIUREWICZ ON HIS 60TH BIRTHDAY

BĘDLEWO, POLAND, JUNE 30 – JULY 5, 2008

## Is chaos compatible with uniform distribution (preliminary report)

Anatole Katok (Penn State University)

A more technical reformulation of the question in the title is: Can a dynamical system with positive topological entropy be uniquely ergodic?

The answer may depend on the class of systems under consideration. For example, it is positive for both homeomorphisms of compact manifolds (M. Rees) and for symbolic systems (Schwartzbauer).

A really interesting question is one for classical systems, i.e. diffeomorphisms and smooth flows on compact manifolds. This question appeared in my discussions with Michel Herman in the late seventies and it led to two significant works at the time: Herman's example showing that the answer is positive if "uniquely ergodic" is weakened to "minimal", and mine that implies in particular that the answer is negative in dimension two. The latter was based on an application of smooth ergodic theory and non-uniform hyperbolicity.

While I always believed that the answer is negative for smooth systems in any dimension and that the proof should use nonuniform partial hyperbolicity, several attempts to implement this approach failed and the problem looked quite resistant. About a year ago my Ph. D. student Sun Peng started to work on a model problem dealing with non-uniformly hyperbolic skew products over uniquely ergodic systems. Recently he succeeded in proving that such a system always has an invariant measure with atomic conditionals in the fibers. This implies negative answer for skew products. It looks like combining a modification of Peng's method with my old idea may give a key to the old problem in the general setting.