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Continuity of the topological entropy and the topological pressure for piecewise monotone maps on the interval

Peter Raith (Universität Wien)

Let $T : [0,1] \to [0,1]$ be a piecewise monotone map, this means there exists a finite partition $\mathcal{Z}$ of $[0,1]$ into finitely many pairwise disjoint open intervals satisfying $\bigcup_{Z\in\mathcal{Z}} \overline{Z} = [0,1]$ such that $T|_Z$ is continuous and strictly monotone for all $Z \in \mathcal{Z}$. Observe that T need not be continuous at the endpoints of the intervals of monotonicity. Two piecewise monotone maps T and $\widetilde{T}$ are said to be ε-close, if they have the same number of intervals of monotonicity and the graph of $\widetilde{T}$ is contained in an ε-neighbourhood of the graph of T considered as subsets of $\mathbb{R}^2$.

It has been proved by Michał Misiurewicz and Wiesław Szlenk that the topological entropy is lower semi-continuous. If $p(T,f) > \sup_{x\in[0,1]} f(x)$, then the pressure is lower semi-continuous according to a result of Mariusz Urbański. Upper bounds for the jumps up of the entropy have been obtained by Michał Misiurewicz. Similar upper bounds can be also found for the jumps up of the pressure. Conditions equivalent to the upper semi-continuity of the pressure for all continuous f are presented. In the case of continuous piecewise monotone maps one of the equivalent conditions is that no endpoint of an interval of monotonicity (except 0 or 1) is periodic. Furthermore, a condition implying the continuity of the measure of maximal entropy is obtained.

Similar questions are investigated for C^1-maps on the interval. Then unimodal maps are considered. Finally results for monotone mod one transformations are presented.