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Finite-time Lyapunov exponents for maps from periodic windows

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Signatures of transient chaos in the time-evolution of finite-time estimates of Lyapunov exponents for maps $f : [a, b] \to [a, b]$, generating attracting limit cycles $(\tilde{x}_j)_{j=1}^{p}$ of periods p are investigated. Finite-time Lyapunov exponent

$$\overline{\lambda}(\rho, n) = \lim_{K\to\infty} K^{-1} \sum_{k=1}^{K} \lambda(n, x_0(k)) = \lim_{K\to\infty} K^{-1} n^{-1} \sum_{k=1}^{K} \log |(f^n)'(x_0(k))|,$$

averaged over a set of map-generated trajectories with initial points $x_0(k)$, distributed according to the density ρ, is the main subject of such investigations. It is conjectured that $\overline{\lambda}(\rho^{\mathrm{inv}}, n)$, where ρ^{inv} is the quasi-invariant measure-density generated by the map, can be approximated by the linear combination $\alpha(n)\lambda_{\mathrm{ch}} + (1 - \alpha(n))\lambda$, in which the “chaotic Lyapunov exponent”, defined by the formula $\lambda_{\mathrm{ch}} = \int_a^b \log |f'(x)|\rho^{\mathrm{qinv}}(x)\mathrm{d}x$, and the standard Lyapunov exponent $\lambda = p^{-1} \sum_{j=1}^{p} \log |f'(\tilde{x}_j)|$ are constant, and only the coefficient α depends on n. What's more, the dependence can be derived from the population dynamics of a set of rambling trajectories, with initial points $x_0(k)$ distributed according to the density ρ^{qinv}. Results of numerical verification of the conjecture are reported.