Conformal Structures and Dynamics (CODY)
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TITLES AND ABSTRACTS OF TALKS

Jon Aaronson (Tel Aviv University)

Relative complexity of random walks in random sceneries

Relative complexity measures the complexity of a probability preserving transformation relative to a factor. We prove distributional limit theorems for the relative complexity of random walks in random sceneries whose associated random walks are α-stable ($1 < \alpha \leq 2$). The results give invariants for `relative isomorphism` of such.

Viviane Baladi (CNRS)

Towards linear response for differentiable unimodal maps with controlled recurrence

(joint work with D. Smania)

When a smooth one-parameter family f_t of dynamical systems admits for all (or many) t a unique SRB measure μ_t, it is natural to ask if the map $t \mapsto \mu_t$ is also smooth. David Ruelle solved the case when the f_t are smooth uniformly hyperbolic, and, more recently, he proposed a formal series as a candidate for the linear response formula (the formula for the derivative of μ_t with respect to t). Suitable resummations of this series have been proved to give the linear response in the one-dimensional piecewise expanding (Baladi-Smania) and analytic nonrecurrent (Ruelle) cases (under a horizontality condition).

In the proofs, the solution α to the twisted cohomological equation (TCE)

$$v(x) = \alpha(f(x)) - f'(x)\alpha(x)$$

(v a smooth "horizontal" function) plays a key part. In the essentially hyperbolic cases solved up to now, it is easy to show that the TCE has a bounded solution. The Collet-Eckmann situation is much more difficult. Recently, we were able to show, under an additional Benedicks-Carleson assumption, that the TCE admits a unique bounded solution, which is continuous. Under the same kind of assumptions we can give a precise description of the spectrum of the transfer operator. We shall explain the ideas in the proofs, and how we expect to exploit this information to obtain linear response (in order to get uniform constants, a stronger, but still generic, recurrence condition will probably be needed).

Balazs Barany (Polish Academy of Sciences)

On the Hausdorff dimension of a family of self-similar sets with complicated overlaps

The main question of the talk is "What is the Hausdorff dimension of the attractor of the iterated function system $\{\gamma x, \lambda x, \lambda x + 1\}$?" The difficulty of the question is that two functions have common fixed point, there are very complicated overlaps, therefore it is not possible to directly apply known techniques. However, we are able to show a formula of the dimension for Lebesgue almost every parameters (γ, λ). Moreover, we show a formula in the special case when $\gamma = \lambda^{p/q}$ for Lebesgue almost every λ.

Krzysztof Barański (University of Warsaw)

Hyperbolic dimension of Julia sets of meromorphic maps with logarithmic tracts

(joint work with B. Karpińska and A. Zdunik)

We prove that for meromorphic maps with logarithmic tracts (in particular, for transcendental maps in the class $\mathcal{B}$, which are entire or meromorphic with a finite number of poles), the Julia set contains a compact invariant hyperbolic Cantor set of Hausdorff dimension greater than 1. Hence, the hyperbolic dimension of the Julia set is greater than 1.

Xavier Buff (Université Paul Sabatier)

Arithmetical condition for non locally connected Julia sets

Dmitry Chelkak (St. Petersburg State University)

Random interfaces in the conformally invariant lattice models and SLE

In this very introductory talk we will discuss
(a) construction of SLE (Schramm-Loewner Evolution) curves;
(b) Schramm's principle which claims that this (one real parameter) family of random curves contains all possible conformally invariant scaling limits

of discrete interfaces coming from (conjecturally or proven conformally invariant) lattice models;
(c) martingale principle which allows one to identify the limiting SLE curve using the only one martingale (with respect to the growing curve) observable, which is a conformally covariant function defined for each simply-connected domain;
(d) how to prove the convergence of interfaces starting with some discrete holomorphic (or discrete harmonic) martingale observable defined for some particular model.

Dmitry Chelkak (St. Petersburg State University)

Universality and conformal invariance in the 2D Ising model

(joint work with S. Smirnov)

We prove that for a large family of planar graphs (so-called isoradial graphs, or, equivalently, rhombic lattices) the Ising model at criticality has a conformally invariant scaling limit which is independent on the underlying graph. Namely, we construct discrete holomorphic fermionic observables for the spin and random cluster representations, show that they converge to conformally covariant scaling limits, and deduce that interfaces converge to Schramm's SLE(3) and SLE(16/3) curves, respectively.

One particular feature of our approach is that we use no deep results about the Ising models: the uniform convergence of observables is deduced using only the discrete analyticity and boundary conditions, universally for arbitrary isoradial graphs.

Arnaud Cheritat (Université Paul Sabatier)

Siegel disks with non locally connected boundary

Van Thomas Cyr (Penn State University)

Obstructions to transient Markov shifts

Neil Dobbs (KTH, Stockholm)

A random walk in exponential dynamics

(joint work with B. Skorulski)

Results concerning ergodic properties of maps from the exponential family $f_\lambda : z \mapsto \lambda \exp(z)$ for which the Julia set is the entire complex plane were limited to two classes: Misiurewicz maps, where the orbit of zero is bounded, and maps for which the orbit of zero grows extremely fast. By modelling distributions of successive returns to the left half plane by a random walk, we can view and generalise these results in a common framework.

Oleksiy Alfredovich Dovgoshey
(National Academy of Science of Ukraine)

Ultrametricity conditions in tangent spaces to metric spaces

We describe the necessary and sufficient conditions under which all tangent spaces at the point of general metric space are ultrametric.

Bertrand Duplantier (CEA)

A rigorous perspective on Liouville quantum gravity & KPZ

(joint work with S. Sheffield)

Polyakov first understood in 1981 that the summation over random Riemannian metrics involved in transition amplitudes in gauge theory or string theory could be represented mathematically by the now celebrated *Liouville theory of quantum gravity.* The quantum gravity measure is formally defined by $d\mu_\gamma = e^{\gamma h(z)} dz$, where dz is the 2D Euclidean (i.e., Lebesgue) measure; $e^{\gamma h(z)}$ is the *random* conformal factor of the Riemannian metric, with h an instance of the Gaussian free field (GFF) on a bounded domain D; and γ is a constant, $0 \leq \gamma < 2$. Outstanding open problems include the relation of Liouville quantum gravity to discrete lattice models and their critical continuum limit — like the Stochastic Schramm-Loewner Evolution — through their embedding in random lattices.

In 1988, Knizhnik, Polyakov and Zamolodchikov predicted that corresponding critical exponents (x) of a conformally invariant statistical model in the Euclidean plane and in Liouville quantum gravity (Δ) would obey the universal *"KPZ relation"*

$$x = \frac{\gamma^2}{4}\Delta^2 + \left(1 - \frac{\gamma^2}{4}\right)\Delta.$$

We present a (mathematically rigorous) probabilistic and geometrical proof of this relation. It uses the properly regularized quantum measure $d\mu_{\gamma,\varepsilon} := \varepsilon^{\gamma^2/2} e^{\gamma h_\varepsilon(z)} dz$, where $h_\varepsilon(z)$ denotes the mean value on the circle of radius ε centered at z of GFF h. When $\varepsilon \to 0$, this measure has both a limit, the Liouville quantum measure, and a Brownian representation in time $t = -\log \varepsilon$, of which KPZ appears as a martingale or large deviations property. The singular case $\gamma > 2$ is also shown to be related to the quantum measure $d\mu_{\gamma'}$, $\gamma' < 2$, by the fundamental duality $\gamma\gamma' = 4$.

Adam Epstein (University of Warwick)

Quadratic differentials, transversality, and the instability of Herman rings

The formalism relating singularities of meromorphic quadratic differentials to the count of nonrepelling cycles admits an extension which accomodates Herman rings. Recent work with Xavier Buff has allowed us to extend the associated variational theory as well. In particular, we make rigorous sense of the notion that there should be some 'complex rotation number' quantity with nonvanishing derivative.

Gernot Greschonig (Tel Aviv University)

On real extensions of distal minimal homeomorphisms

The talk presents a structure theorem for topologically conservative real skew product extensions of a distal minimal homeomorphism on a compact metric space. Our main result states that every such extension can be represented by a perturbation of a Rokhlin skew product.

Moreover, we give certain counterexamples to point out that all components of the construction, including the perturbation, are in fact inevitable.

Yonatan Gutman (The Hebrew University of Jerusalem)

Embedding $\mathbb{Z}^k$-actions in cubical shifts and $\mathbb{Z}^k$-symbolic extensions

Joanna Jaroszewska (University of Warsaw)

On Hausdorff dimension of invariant measures of weakly contracting on average iterated function systems

(joint work with M. Rams)

Bogusława Karpińska (Warsaw University of Technology)

Dimension properties of the boundaries of exponential basins

(joint work with K. Barański and A. Zdunik)

In this talk we will prove that the boundary of a component U of an attracting periodic basin (of period at least 2) for an exponential map has Hausdorff dimension greater than 1 and less than 2. We will also discuss the role of the sets of escaping and non-escaping points in ∂U in terms of the Hausdorff dimension.

Iurii Sergeevich Kolomoitsev
(National Academy of Science of Ukraine)

Some features of the space L_p, $0 < p < 1$

The space $L_p, 0 < p < 1$, is essentially different from the space L_p, $p \geq 1$. Some new features of this space will be presented. In particular, we will consider the following problems:

1) Completeness of trigonometric systems with the gaps.
2) Comparison of differential operators.
3) The non-existing of Fourier multipliers.

Pekka Johannes Koskela (University of Jyväskylä)

Planar mappings of finite distortion: geometric questions

No geometric characterization is known for images of the unit disk under homeomorphisms of the plane that have locally exponentially integrable distortions. I will explain what is currently known.

Janina Kotus (Warsaw University of Technology)

On the Hausdorff dimension of the escaping set of certain meromorphic functions

(joint work with W. Bergweiler)

Let f be a transcendental meromorphic function of finite order ρ for which the set of finite singularities of f^{-1} is bounded. Suppose that ∞ is not an asymptotic value and that there exists $M \in \mathbb{N}$ such that the multiplicity of all poles, except possibly finitely many, is at most M. For $R > 0$ let $I_R(f)$ be the set of all $z \in \mathbb{C}$ for which $\liminf_{n\to\infty} |f^n(z)| \geq R$ as $n \to \infty$. Here f^n denotes the n-th iterate of f. Let $I(f)$ be the set of all $z \in \mathbb{C}$ such that $|f^n(z)| \to \infty$ as $n \to \infty$; that is, $I(f) = \bigcap_{R>0} I_R(f)$. Denote the Hausdorff dimension of a set $A \subset \mathbb{C}$ by $\mathrm{HD}(A)$. It is shown that $\lim_{R\to\infty} \mathrm{HD}(I_R(f)) \leq 2M\rho/(2+M\rho)$. In particular, $\mathrm{HD}(I(f)) \leq 2M\rho/(2+M\rho)$. These estimates are best possible: for given ρ and M we construct a function f such that $\mathrm{HD}(I(f)) = 2M\rho/(2+M\rho)$ and $\mathrm{HD}(I_R(f)) > 2M\rho/(2+M\rho)$ for all $R > 0$.

If f is as above but of infinite order, then the area of $I_R(f)$ is zero. This result does not hold without a restriction on the multiplicity of the poles.

Genadi Levin (Hebrew University of Jerusalem)

Sequences of "satellite" renormalizations

We give quite weak combinatorial conditions under which the map at the limit of such renormalizations is rigid and its Julia set is not locally connected.

Ian Melbourne (University of Surrey)

Decay of correlations for nonuniformly hyperbolic systems (revisited)

Sarig 2002 and Gouezel 2004 used operator renewal sequences to study rates of decay of correlations for systems modelled by a Young tower. (In the noninvertible setting, this is a Markov map with a Gibbs-Markov induced map.) The decay rate is determined by the tails of the return time. In certain situations (including polynomial and stretched exponential rates) their results are optimal.

I will describe a significantly more elementary approach (work in progress with Dalia Terhesiu) which applies for general return times. It recovers many, perhaps all, of the previous results, as well as strengthening some of them. It also provides a new approach for proving statistical limit theorems.

Pierre Nolin (New York University)

Connection probabilities and RSW-type bounds for the FK Ising model

For two-dimensional percolation, Russo-Seymour-Welsh bounds on crossing probabilities are an important a-priori indication of conformal invariance – or at least scale invariance. They turned out to be instrumental to describe critical and near-critical percolation, they are for instance a key tool to derive the so-called scaling relations, that link the critical exponents associated with the main macroscopic functions.

We prove Russo-Seymour-Welsh-type uniform bounds on crossing probabilities for the FK Ising model at criticality, independent of the boundary conditions. A crucial tool in our proof is Smirnov's fermionic observable for the FK Ising model, that makes appear some harmonicity on the discrete level, allowing to get precise estimates on boundary connection probabilities.

Our proof remains purely discrete (in particular we do not make use of any continuum limit), and it allows to derive directly several noteworthy results – some new and some not – among which the fact that there is no magnetization at criticality, tightness properties for the interfaces and the value of the half-plane one-arm exponent.

Tomas Persson (Polish Academy of Sciences)

Dimension of piecewise hyperbolic attractors with overlaps

Carsten Lunde Petersen (Roskilde Universitet)

Surveying the cubic connectedness locus

Several authors including Milnor, Epstein and Yampolsky, Zakeri, Roesch, Avila, Lyubich and Shen, Tan Lei and myself have described different aspects of the connectedness locus C of cubic polynomials. In this talk I will merge most of these aspects into a limb description of the cubic connectedness locus. This readily gives descriptions of huge chuncks of C and is lends itself to Yoccoz-type puzzle constructions, which promises for further understanding.

Michał Rams (Polish Academy of Sciences)

Lyapunov spectrum for rational maps

(joint work with K. Gelfert and F. Przytycki)

I will present our recent work in which we calculate the Lyapunov spectrum, i.e. the Hausdorff dimension of the set of points $x \in J$ for which the Lyapunov exponent exists and equals α, as a function of α.

Juan Rivera-Letelier (Pontificia Universidad Católica de Chile)

Equivalences of non-uniform hyperobilicity conditions for S-multimodal maps

A transitive S-multimodal map is said to be uniformly hyperbolic on periodic points if the infimum of the Lyapunov exponents of periodic points is strictly positive. We show that a transitive S-multimodal map is uniformly hyperbolic on periodic points if, and only if, it has an absolutely continuous invariant measure with exponential decay of correlations. The equivalence of several other non-uniform hyperbolicity conditions, such as the topological Collet-Eckmann condition, follow. We thus extend to S-multimodal maps previous results of Nowicki and Sands, and Nowicki and Przytycki for

S-unimodal maps. The proofs are based in part on analogous results for complex rational maps, obtained in a previous joint work with Przytycki and Smirnov.

Ruslan Salimov (National Academy of Science of Ukraine)

On the bi-Sobolev homeomorphisms

Recall that, given a family of paths Γ in $\mathbb{R}^n$, a Borel function $\varrho : \mathbb{R}^n \to [0,\infty]$ is called **admissible** for Γ, abbr. $\varrho \in adm\,\Gamma$, if

$$\int\limits_{\gamma} \varrho\, ds \ \geq \ 1 \tag{1}$$

for all $\gamma \in \Gamma$. The (conformal) **modulus** of Γ is the quantity

$$M(\Gamma) \ = \inf_{\varrho \in adm\,\Gamma} \int\limits_{G} \varrho^n(x)\ dm(x)\ . \tag{2}$$

A homeomorphism $f : G \to G'$ is said to be $W^{1,n}_{loc}$-**bi-Sobolev map** if f belongs to the Sobolev space $W^{1,n}_{loc}(G,G')$ and its inverse f^{-1} belongs $W^{1,n}_{loc}(G',G)$.

Let G and G' be domains in $\mathbb{R}^n$, $n \geq 2$, and let $Q_1 : G \to [1,\infty]$ be a measurable function and $Q_2 : G' \to [1,\infty]$ be a measurable function. A homeomorphism $f : G \to G'$ is called a $(Q_1(x),\, Q_2(y))-$ **homeomorphism** if

$$M(f\Gamma) \leq \int\limits_{G} Q_1(x)\cdot \varrho^n(x)\ dm(x) \tag{3}$$

$$M(\Gamma) \ \leq \int\limits_{D_*} Q_2(y)\cdot \rho_*^n(y)\ dm(y) \tag{4}$$

for every family Γ of paths in G and $\rho \in adm\,\Gamma$ and $\rho_* \in adm\,f\Gamma$.

Theorem. Let G and G' be domains in $\mathbb{R}^n$, $n \geq 2$, and $f : G \to G'$ is $(Q_1(x), Q_2(y))$–homeomorphism with $Q_1 \in L^1_{loc}(G)$ and $Q_2 \in L^1_{loc}(G')$. Then f is $W^{1,n}_{loc}$- bi-Sobolev map.

Duncan Sands (CNRS / Université Paris-Sud 11)

The Schwarzian derivative in one-dimensional dynamics

Omri Moshe Sarig (Weizmann Institute of Science)

Spectral gap for Ruelle operators

Yevgen Sevostyanov (National Academy of Science of Ukraine)

Absolute continuity on lines of one class of space mappings

Let D be a domain in $\mathbb{R}^n$, $n \geq 2$. A mapping $f : D \to \mathbb{R}^n$ is said to be *discrete* if the preimage $f^{-1}(y)$ of every point $y \in \mathbb{R}^n$ consists of isolated points, and an *open* if the image of every open set $U \subseteq D$ is open in $\mathbb{R}^n$. In what follows $\overline{\mathbb{R}^n} = \mathbb{R}^n \cup \{\infty\}$. Recall that a Borel function $\rho : \mathbb{R}^n \to [0, \infty]$ is said to be *admissible* for family Γ of paths γ in $\mathbb{R}^n$, if $\int\limits_{\gamma} \rho(x) \quad |dx| \geq 1$ for all paths $\gamma \in \Gamma$. In this case we write $\rho \in \operatorname{adm} \Gamma$. The *modulus* $M(\Gamma)$ of Γ is defined as

$$M(\Gamma) = \inf_{\rho \in \operatorname{adm} \Gamma} \int\limits_{\mathbb{R}^n} \rho^n(x) dm(x)$$

interpreted as $+\infty$ if $\operatorname{adm} \Gamma = \varnothing$. Given a domain D and two sets E and F in $\overline{\mathbb{R}^n}$, $n \geq 2$, $\Gamma(E, F, D)$ denotes the family of all paths $\gamma : [a, b] \to \overline{\mathbb{R}^n}$ which join E and F in D, i.e., $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in D$ for $a < t < b$. Let $r_0 = \operatorname{dist}(x_0, \partial D)$ and $Q : D \to [0, \infty]$ be a measurable function. Set

$$A(r_1, r_2, x_0) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\},$$

$$S_i = S(x_0, r_i) = \{x \in \mathbb{R}^n : |x - x_0| = r_i\}, \quad i = 1, 2.$$

A mapping $f : D \to \overline{\mathbb{R}^n}$ is said to be a *ring Q–mapping at a point $x_0 \in D$*, if

$$M\left(f\left(\Gamma\left(S_1,\, S_2,\, A\right)\right)\right) \leq \int\limits_{A} Q(x) \cdot \eta^n(|x - x_0|)\, dm(x) \tag{5}$$

holds for every annulus $A = A(r_1, r_2, x_0)$, $0 < r_1 < r_2 < r_0$ and every measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that

$$\int\limits_{r_1}^{r_2} \eta(r)\, dr \geq 1.$$

We say that a continuous sense–preserving mapping $f : D \to \overline{\mathbb{R}^n}$ is a *ring $Q-$mapping in D* if (5) holds for every $x_0 \in D$.

Theorem 1. *Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, and $f : D \to \overline{\mathbb{R}^n}$ be a ring Q–mapping with $Q \in L^1_{loc}$. Suppose that f is discrete and open. Then f is differentiable a.e. in D.*

Theorem 2. *Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, and $f : D \to \overline{\mathbb{R}^n}$ be a ring Q–mapping with $Q \in L^1_{loc}$. Suppose that f is discrete and open. Then $f \in ACL$.*

Theorem 3. *Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, and $f : D \to \overline{\mathbb{R}^n}$ be a ring Q–mapping with $Q \in L^1_{loc}$. Suppose that f is discrete and open. Then f has $\left(N^{-1}\right)$–property.*

Weixiao Shen (National University of Singapore)

Statistical properties of one-dimensional maps with weak hyperbolicity

Sebastian van Strien (University of Warwick)

Quasi-symmetric rigidity of real analytic one-dimensional maps

Grzegorz Michał Świrszcz (IBM Research)

Invariant sets for 2-dimensional dynamical systems defined by piecewise isometries

Xavier Tolsa (ICREA / Universitat Autònoma de Barcelona)

Quasiconformal maps, distortion of Hausdorff measures, and the Painlevé problem

In the quasiconformal setting, the Painlevé problem consists in characterizing removable sets for bounded K-quasiregular functions. To solve this problem, it is important to know how Hausdorff measures and analytic capacity behave under quasiconformal mappings. Also, Riesz capacities associated to non linear potentials arise naturally. In this talk I will review recent results on this topic.

Mariusz Urbański (University of North Texas)

Random distance expanding maps

In this talk we introduce measurable expanding random systems and discuss the thermodynamical formalism and establish, in particular, exponential decay of correlations and real analyticity of the expected pressure despite the fact that the spectral gap property does not hold.

This thermodynamic formalism will be then used to investigate fractal properties of conformal random systems. It includes Bowen's formula and the multifractal formalism of the Gibbs states.

Depending on the behavior of the Birkhoff sums of the pressure function we get a natural classifications of the systems into two classes: quasi-deterministic systems which share many properties of deterministic ones and essential random systems which are rather generic and never bi-Lipschitz equivalent to deterministic systems. In the essential case that the Hausdorff measure vanishes which refutes a conjecture of Bogenschütz and Ochs. We finally give applications of our results to various specific conformal random systems and positively answer a question of Brück and Büger concerning the Hausdorff dimension of random Julia sets.

Yuki Yayama (Universidad de Chile)

Existence of a measurable saturated compensation function between subshifts and its applications

We show the existence of a bounded Borel measurable compensation function for a factor map between subshifts. As an application, we consider an expanding nonconformal map on the torus given by an integer-valued diagonal matrix and give a formula for the Hausdorff dimension for a compact invariant set represented by a subshift and characterize the invariant ergodic measures of full dimension. We also study uniqueness of the measure of full dimension.

Anna Zdunik (University of Warsaw)

Endomorphisms of complex projective spaces – ergodic theory

Michel Zinsmeister (Université d'Orléans)

Regularized growth processes