Modern Dynamics and its Interaction with Analysis, Geometry and Number Theory

Workshop, July 17-23, 2011

Titles and Abstracts

1. Alexander I. Bufetov (The Steklov Institute of Mathematics, Moscow),
   On the Vershik-Kerov Conjecture Concerning the Shannon-McMillan-Breiman Theorem for the Plancherel Family of Measures on the Space of Young Diagrams

   Vershik and Kerov conjectured in 1985 that dimensions of irreducible representations of finite symmetric groups, after appropriate normalization, converge to a constant with respect to the Plancherel family of measures on the space of Young diagrams. The statement of the Vershik-Kerov conjecture can be seen as an analogue of the Shannon-McMillan-Breiman Theorem for the non-stationary Markov process of the growth of a Young diagram. The limiting constant is then interpreted as the entropy of the Plancherel measure. The talk is devoted to the proof of the Vershik-Kerov conjecture. The argument relies on the methods of Borodin, Okounkov and Olshanski. The talk is based on the preprint `http://arxiv.org/abs/1001.4275`

2. Van Cyr (Northwestern University)
   A number theoretic question arising in the geometry of plane curves and in billiard dynamics

   The topic of this talk is a number theoretic conjecture that has arisen recently in the works of E. Gutkin (in connection with billiard dynamics) and S. Tabachnikov (in connection with "bicycle curves"): if $n > 1$ is an integer, then the only rational solution to

   $$n \tan(\pi x) = \tan(n\pi x)$$

   is $x = 1/2$. In a recent paper of mine, I showed that this is true. In this talk, I will discuss how this question arises and how it is proven.

3. Vincent Delecroix (Institut de Mathématiques de Luminy)
   Diffusion in the periodic wind-tree model

   The periodic wind-tree model (or rectangular periodic Lorenz gas) is a billiard in the Euclidean plane in which identical rectangular scatterers are regularly centered at each point of the integer lattice. We prove that, independently of the size of the scatterers, generically with respect to the angle, the polynomial diffusion rate is 2/3; in other words the maximal distance reached by a particle in the billiard before time t is roughly $t^2/3$.

4. Alexander Felshtyn (Szczecin University)
   Topological entropy, the growth rate of Floer homology and symplectic zeta function

   The main theme of this talk is to compute for a symplectomorphism $\phi : M \to M$ of a compact surface, the asymptotic invariant $F_\infty(\phi)$ which is defined to be the growth rate of the sequence $\dim HF(\phi^n)$ of the total dimensions of symplectic Floer homologies of the iterates of ϕ. We prove that the asymptotic invariant coincides with the largest dilatation of the pseudo-Anosov components of ϕ and its logarithm coincides with the topological entropy . This implies that the symplectic zeta function of ϕ has a positive radius of convergence.

5. Krzysztof Frączek (Nicolaus Copernicus University)
Ergodic properties of infinite measure-preserving extensions of area-preserving flows

I will present some methods for proving ergodicity of skew product extensions of interval exchange transformations of periodic type. I will also deal with a class of smooth ows on non-compact manifolds which are extensions of so called multivalued Hamiltionian ows on compact surfaces of higher genus. These ows have Poincare' sections for which the first recurrence map is isomorphic to a skew product of an IET and a BV cocycle or a cocycle with logarithmic singularities. This allows us to prove a sufficient condition for ergodicity whenever the IET has periodic type. My talk will be based on two papers joint with J.-P. Conze and C. Ulcigrai.

6. Światosław Gal (Uniwersytet Wrocławski and Universität Wien)
$d\mathfrak{K}(g) = g^*\alpha - \alpha$.

Let G be an abstract group acting on a space X. Let g be an element of G. It is a vague, but interesting problem what could be deduced about group properties of $g \in G$ from the dynamical properties of the action of g.

In the talk we would construct a natural cocycle on a class of groups of homeomorphisms. This class includes, among others, groups of Hamiltonian diffeomorphisms of symplectically aspherical manifolds and groups of homeomorphisms of a space preserving given first cohomology class. We will use this cocycle to prove that certain elements cannot be distorted (in other terms they have nonzero translation length).

As a side remark we will deduce that fundamental group of symplectically hyperbolic manifold cannot be amenable.

This is a joint work with Jarek Kędra from U. of Aberdeen.

7. Alexander Gorodnik (University of Bristol)
Mixing for higher-rank abelian actions on nilmanifolds

We establish mixing and mixing of all orders for actions of commutative groups of automorphisms on nilmanifolds. Our argument is based the theory of Diophantine approximation developed by W. Schmidt and A. Baker, and allows to establish quantitative estimates on decay of correlations. This is a joint work with R. Spatzier.

8. Mike Hochman (Princeton University),
Slow entropy invariants and differentiable models for infinite-measure preserving Z^k actions

By a classical theorem of Krengel, if a map preserves an infinite, sigma-finite measure, then it is measurably isomorphic to a diffeomorphism of a compact manifold preserving a Borel measure. Of course, the analogous statement for finite measures is false: infinite entropy maps do not have such models. This difference underscores the fact that there is no good notion of entropy in the infinite-measure setting (in fact there are at least four invariants called "entropy", but none has a useful geometric or information-theoretic interpretation).

In the higher rank case, however, it turns out that there do exist infinite measure Z^d actions, $d > 1$, which do not have smooth models in the above sense. The obstruction involves a "slow entropy" invariant similar to the Katok and Thouvenot invariant, with a number of differences. I will discuss this result and the relevant background.

9. Huyi Hu (Michigan State University)
Essential coexistence of chaotic and nonchaotic behavior in a volume preserving diffeomorphism

We show that there exists a smooth volume preserving topologically transitive diffeomorphism of a compact smooth Riemannian manifold which is ergodic and has nonzero Lyapunov exponents on an open and dense subset of not full measure and has zero Lyapunov exponent on the complement. This confirms "essential" coexistence of "completely" chaotic and "absolutely" non-chaotic behavior in the class of smooth volume preserving systems.

This is a joint work with Ya. Pesin and A. Talitskaya.

10. Shirali Kadyrov (University of Bristol)
Entropy and escape of mass for diagonal actions

We study the relation between entropy and escape of mass in the non-compact homogeneous spaces. We mainly consider the action T of a particular diagonal element on the space of 3-lattices. We prove that any limit measure of a sequence of T-invariant probability measures with high entropy cannot be zero. The idea behind the proof recovers Y. Cheung's result on the Hausdorff dimension of singular vectors in the plane.

11. Anatole Katok (Penn State University)
The new notion of entropy for actions of higher rank abelian groups, and its connections to slow entropy and rigidity

Since ordinary measure-theoretic entropy for a smooth measure preserving action of any countable group other that cyclic or its finite extension vanish, alternative notions of entropy for such actions are of interest. In particular, for a smooth action of Z^k slow entropy based on the scale function $n^{1/k}$ provides a proper normalization and gives a first cut into 'zero entropy" and 'positive entropy" actions. However, in order to ascribe a numerical value to slow entropy one needs to fix a norm on the acting group and this (unlike fixing a 'volume element") is somewhat arbitrary.

In this talk I will discuss a new and natural notion of average entropy that is equal to the inverse of the volume of the unit ball in the entropy norm. Thus is it positive if and only if all non-identity elements of the action have positive entropy. Average entropy is equal to the infimum of the values of the $n^{1/k}$ slow entropy over all norms on the acting group normalized to the volume element. If it is positive, the infimum is achieved at the entropy norm. A corollary of strong rigidity results that are discussed in Federico Rodriguez Hertz's talk is that for the maximal rank actions (where the rank is at least two and dimension is greater then rank by one), the average entropy is either equal to zero or is bounded from below by a positive number that depends only on the dimension. Conjecturally this bound is uniform in dimension. This is a joint work in progress with Federico Rodriguez Hertz.

12. Svetlana Katok (Penn State University)
Reduction theory, coding of geodesics, and continued fractions

I will discuss a method of coding of geodesics on surfaces of constant negative curvature using boundary maps and "reduction theory". For compact surfaces these maps are generalizations of the Bowen-Series map. For the modular surface they are related to a family of (a,b)-continued fractions. In special cases, when an (a,b)-expansion has a so-called "dual", the coding sequences are obtained by juxtaposition of the boundary expansions of the fixed points, and the set of coding sequences is a countable sofic shift. I will also give a dynamical interpretation of the "reduction theory" which underlines these constructions and its relation to the attractor of a certain associated natural extension map. The talk is based on joint works with Ilie Ugarcovici.

13. Feliks Przytycki (IMPAN)
TBA

14. Felipe Ramirez (University of Bristol),
Higher cohomology for Anosov actions

The talk will be on a conjecture due to A. and S. Katok stating that, for any standard partially hyperbolic action by a rank d abelian group, the smooth n-cohomology with real coefficients trivializes for all $1 \leq n \leq d-1$, and the smooth d-cohomology is determined by periodic orbits of the action. In the top degree, the conjecture is a higher-rank counterpart to Livshitz's theorem. In the intermediate degrees, it fits with results of A. Katok and R. Spatzier stating that the first cohomology trivializes for higher-rank partially hyperbolic actions. Katok and Katok proved the conjecture for abelian groups of hyperbolic toral automorphisms. I will focus on the action on $[SL(2,R) \times \cdots \times SL(2,R)]/G$ by the subgroup of diagonal matrices. The main tools are from representation theory.

15. Federico Rodriguez Hertz (Penn State University)
Arithmeticity for some higher rank abelian actions

Let α be a measure preserving higher rank abelian action on a manifold. It turns out that under some conditions on the Weyl chamber picture of the action this numbers are logarithms of algebraic numbers of the correct degree. For example, for Z^2 actions on 3 dimensional manifolds with kernel of Lyapunov functionals in general position, the entropies are are logarithms of algebraic numbers of degree 3. Moreover, as an outcome of the proof we get a very nice description of the possible topological types of the manifold. This is joint work with A. Katok.

16. Andrzej Schinzel (IMPAN)
Ten problems in Number Theory

17. Corinna Ulcigrai (University of Bristol),
Irregular Z-covers of translation surfaces

Recently there has been a lot of interest in the ergodic theory of infinite translation surfaces, in particular Z-covers of compact translation surfaces. The study of directional flows on such surfaces is related to Z-valued cocycles over interval exchange transformations. In joint work with K. Frączek, we exhibit a class of Z-covers for which the directional flow is not ergodic for a.e. direction. Moreover, the cocycles induced as Poincare' maps are irregular.

18. Zhenqi Wang (Yale University)
Rigidity of algebraic partially hyperbplic actions

Geometric method is succesfully used to get rigidity of genetric Cartan actions on homegeneous space generated by quasi-split groups. For general actions, to apply KAM method how to get splitting is problematic. I will show how to use orthogonal projections to get over this difficulty.

19. Bryce Weaver
Growth Rate of Periodic Orbits for a Class of Non-Uniformly Hyperbolic Flows

Using some verifiable properties and local analysis, we are able to construct a Margulis measure on a class of 3-dimensional non-uniformly hyperbolic geodesic flows, constructed by V. Donnay. The class of metrics can be applied to any surface, in particular S^2. This measure is used to obtain precise asymptotics of the growth rate of periodic orbits of the form,

$$P(t) \sim \frac{e^{ht}}{ht},$$

for *dsh* equal to the topological entropy and $P(t)$ is the number of periodic orbits of period at most t.