

ALEXANDER GORODNIK
BASICS OF LIE THEORY, ALGEBRAIC AND ARITHMETIC
GROUPS FOR DYNAMICISTS

Week 1: 27 June - 2 July, 5 double lectures (each lecture is 45-50 minutes).

I. Basic notions of Lie theory:

- Basic examples and definitions of Lie theory
- Lie algebra, exponential map, and Lie correspondence
- Nilpotent, solvable and semisimple groups
- Cartan subgroups and root systems
- Homogeneous spaces
- Integration and invariant measures

II. Introduction to algebraic groups

- Algebraic varieties and algebraic groups
- Semisimple/unipotent elements and Jordan decomposition
- Representations of SL_2
- Rigidity of algebraic actions

III. Lattices and arithmetic groups

- Some geometric and algebraic constructions of lattices
- Notions of commensurability and irreducibility
- Mahler and Godement compactness criteria
- Borel density theorem
- Introduction to rigidity of arithmetic groups

References:

1. Y. Benoist, Five lectures on lattices in semisimple Lie groups <http://www.math.u-psud.fr/~benoist/prepubli/prepublication.html>
2. J.S. Milne, Algebraic Groups, Lie Groups, and their Arithmetic Subgroups <http://www.jmilne.org/math/CourseNotes/ala.html>
3. W. Rossmann, Lie Groups: An Introduction Through Linear Groups
4. J.-P. Serre, Lie Algebras And Lie Groups
5. D. Witte Morris, Introduction to arithmetic groups <http://people.uleth.ca/~dave.morris/books/IntroArithGroups.html>