

OMRI SARIG

INTRODUCTION TO ERGODIC THEORY

Week 1: 27 June - 2 July, 5 double lectures (each lecture is 45-50 minutes).

**Abstract:** The course will give a rapid overview of the ergodic theoretic tools which are most likely to be used in the advanced minicourses given in the second and third week (with the exception of entropy, which is covered in Ledrappier's course).

**Part 1:** Basic definitions and basic constructions

- (1) Probability preserving maps
- (2) Ergodicity, weak mixing, mixing
- (3) Basic constructions
  - (a) Krylov-Bogolyubov construction
  - (b) Products and skew-products
  - (c) Factors
  - (d) Suspensions
  - (e) Induced transformations

**Part 2:** The pointwise ergodic theorem

- (1) Statement and proof
- (2) Conditional expectations and the identification of the limit
- (3) Conditional probabilities and the ergodic decomposition

**Part 3:** The multiplicative ergodic theorem

- (1) Kingman's subadditive ergodic theorem (statement) and applications
- (2) Statement of the Multiplicative ergodic theorem
- (3) Derivation from Karlsson-Margulis ergodic theorem for isometric actions

**Part 4:** Proof of the Karlsson-Margulis ergodic theorem

- (1) Horofunctions and Busemann functions
- (2) Karlsson-Ledrappier strategy
- (3) Proof

**Part 5:** Basic notions from spectral theory (as much as time allows)

- (1) Koopman operator and spectral isomorphism
- (2) Spectral measures
- (3) Discrete spectrum, continuous spectrum, Lebesgue spectrum