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Abstracts of contributed talks and posters

FINAL VERSION

Local classification of singular hexagonal 3-webs with holomorphic Chern connection and infinitesimal symmetries.

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Abstract

A finite collection of foliations form a web. Blaschke discovered that already for a 3-web in the plane, there is a nontrivial local invariant, namely the curvature form. Thus any local classification of 3-webs necessarily has functional moduli if no restriction on the class of webs is imposed. The most symmetric is a hexagonal 3-web when the curvature is supposed to vanish identically. In a regular point a hexagonal 3-web is locally diffeomorphic to 3 families of parallel lines. For singular points, where at least two foliations are not transverse, two hexagonal 3-webs are not necessarily locally diffeomorphic. We provide a complete classification of hexagonal singular 3-web germs in the complex plane, satisfying the following two conditions:

1) the Chern connection form remains holomorphic at the singular point,
2) the web admits at least one infinitesimal symmetry at this point.

As a by-product, classification of hexagonal weighted homogeneous 3-webs is obtained.

Key words: hexagonal 3-web, implicit ODE, Chern connection, infinitesimal symmetries.

AMS Subject classification: 53A60 (primary), 34M35, 37C80 (secondary).

POSTER

Global dynamics for planar maps in the presence of symmetry

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Abstract

This poster is a joint work with I. Labouriau and S. B. S. D. Castro of CMUP and deals with the global dynamics of a planar map which has a unique fixed point. In the case that the map is equivariant by the action of a compact Lie group, it is possible to describe the local dynamics and -from this - also the global dynamics. In particular, if the group contains a reflection, there is an line invariant by the map. This allow us to use results of B. Alarcon, C. Gutierrez and V. Guiñez based on the Theory of Free Homeomorphisms to describe the global behavior of the solutions. In the absence of reflections, we show some examples which are also equivariant and the local stability of the unique fixed point does not imply global stability. Those examples are motivated by results of R. Ortega and F. R. Ruiz del Portal based on the Theory of Prime Ends.

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POSTER

Azeb Alghanemi (University of Leeds),

Symmetry set and the singularity of the radius function

The symmetry set and its sun medial axis play a central role in many applications such as object recognition , object reconstruction and medical imaging. In this talk I will talk about two main parts. The first one deals with the creating of the symmetry set in $\mathbb{R}^{n+1}$ from its smooth boundary. Also, the necessary and sufficient condition for two points on the boundary to form a symmetry point will be investigated. The second part takes the opposite direction and deals with the creating of the boundary from its symmetry set. Furthermore, the singularity of the radius function will be studied through this part.

Łojasiewicz exponents, Rees mixed multiplicities and Newton filtrations

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Abstract

Given an analytic map germ $g : (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ such that $g^{-1}(0) = \{0\}$ and a Newton polyhedron $\Gamma_+ \subseteq \mathbb{R}^n_+$, we show an upper bound for the Łojasiewicz exponent of g, denoted by $\mathcal{L}_0(g)$, in terms of the Newton filtration induced by Γ_+ and we show a condition on g that implies that $\mathcal{L}_0(g)$ reaches this upper bound. In particular, we obtain a wide class of semi-weighted homogeneous functions $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ for which the Łojasiewicz exponent $\mathcal{L}_0(\nabla f)$ attains the maximum possible value. The techniques that we apply come from multiplicity theory in local rings. In particular, we apply the notion of Rees mixed multiplicities of a set of ideals. Our work has been motivated by a previous result due to T. Krasiński, G. Oleksik and A. Płoski for weighted homogeneous functions $(\mathbb{C}^3, 0) \to (\mathbb{C}, 0)$. This is a joint work with Santiago Encinas (University of Valladolid).

POSTER

Degenerate singularities and their Milnor numbers

Szymon Brzostowski

Abstract

We give an example of a curious behavior of the Milnor number with respect to evolving degeneracy of an isolated singularity in $\mathbb{C}^2$

Intrinsic Complete Transversals and symmetric bifurcations

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(joint work with Andrew du Plessis, U. Aarhus, Denmark)

Let G be a Lie group acting smoothly on an affine space A and let W be a vector subspace of V_A. A vector subspace T of W is a *complete transversal* if it is transversal to the orbit of $x_0 \in A$ and meets each orbit through the affine space $x_0 + W$ of A.

We show how intrinsic complete transversals provide a systematic way both to classify map-germs with respect to natural equivalences and to solve the recognition problem with respect to such a classification.

We apply this method to the classification and recognition of bifurcation problems, with and without symmetry. In many cases a complete transversal can be chosen, which is invariant under a large subgroup of the group of equivalences. Then the recognition problem may be solved systematically.

We illustrate the power of this method by completing the classification of D_4-equivariant bifurcation problems of Golubitsky and Roberts [1], by finding a missing normal form of low codimension.

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On the Geometry and Topology of Transverse Vector Fields

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We consider smoothly embedded, orientable, hypersurfaces in real n-dimensional space. In Euclidean differential geometry there is an invariant transverse vector field, namely the unit normal field, which carries a lot information about the underlying hypersurface. In equi-affine differential geometry, there is an invariant transverse vector field, namely the equi-affine normal field, which carries a lot of equi-affine information about the underlying hypersurface. Likewise in Minkowski and other differential geometries.

In this talk we consider a more general setting where the hypersurface has an arbitrary smooth transverse vector field. We develop the general theory along the lines of its Euclidean, equi-affine and Minkowski counterparts; proving similar results. Working in this more general setting means that we can prove the results for many different geometries at the same time.

We discuss the idea of relative distance and relative height functions. the bifurcations sets of these families can be easily calculated. Although the calculations are simple, they pose difficult problems for the classification of singularities.

The Euclidean, equi-affine and Minkowski normal fields in some sense "belong to" the same class of smooth transverse vector fields. We use this to define a quotient space of the space of smooth transverse vector fields, and then study its topology.

To fully understand, and to put into place, out findings requires us to consider the classification of the Lie subgroups of the projective linear group. This is a famously wild problem, with no known solution for higher dimensions.

On the points realizing the distance to a definable set

Maciej P. Denkowski
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Abstract: The starting point of this lecture is a useful lemma, presumably due to John Nash, concerning the points realizing the Euclidean distance to an analytic submanifold of $\mathbb{R}^n$. We will present a generalization of this result for (a definable family of) sets definable in some o-minimal structure and we discuss the properties of the multifunction obtained.

In other words, for a given definable (or subanalytic) closed set $M \subset \mathbb{R}^n$ we are interested in the the multifunction assigning to each point $x \in \mathbb{R}^n$ the compact set $m(x) \subset M$ of points $y \in M$ realizing $\mathrm{dist}(x, M)$, as well as in the structure of the exceptional set E of points for which $\#m(x) > 1$. This study is closely related to such notions as skeletons, central sets and conflict sets.

Daniel Dreibelbis (University of North Florida)
Duality for immersed manifolds

Let M^n be an immersed manifold in $\mathbb{R}^{2n}$. Because the dimension and co-dimension of M are equal, we have a natural duality between the normal bundle and the tangent bundle. In this talk, we will explore this duality, connecting concepts found in second order geometry, as well relating the singularities of various geometric maps that occur for generic immersions. The focus will be on results that are independent of the dimension of M, with examples given for surfaces in $\mathbb{R}^4$ and 3-manifolds in $\mathbb{R}^6$.

POSTER

Generic affine differential geometry of surfaces in $\mathbb{R}^4$

L. Espinoza-Sánchez

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Abstract

In [3], Izumiya defined two surfaces associated to a curve (intrinsic affine binormal developable, affine rectifying Gaussian surface) and characterized them from the generic singularities of the affine distance and height functions. Davis in [1] studied affine differential geometry of curves in $\mathbb{R}^n$, generalized the affine distance and height functions, studied their singularities and defined equi-affine frames. In [2], Davis considered smoothly embedded hypersurfaces $M \subset \mathbb{R}^{n+1}$ in the context of affine geometry and defined an affine normal curvature which is a differential invariant. Our proposal in this work is to study the geometry of smooth manifolds of codimension two embedded in $\mathbb{R}^n$, generalizing the affine distance and height functions, and studying their generic singularities. We consider here the special case of surfaces embedded in $\mathbb{R}^4$ and apply the theory developed by Nomizu-Vrancken in [4].

This is a work in progress, done jointly with J. J. Nuño-Ballesteros and M.J. Saia, with financial support from FAPESP.

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Discriminants of systems of equations and tropical double point formula

Alexander Esterov

If a proper map F of complex manifolds is generic enough, then homology classes of multisingularities of F can be expressed in terms of topology of F. In the same spirit, if F is a (non-proper) polynomial map of complex tori, and components of F are generic linear combinations of finitely many given monomials, then homology classes of multisingularities of F in a suitable compactification of its range can be expressed in terms of the given collection of monomials. The sources of motivation for the study of such an expression are the same as for the classical multisingularity theory, but efficient tools for this study are expected to be very different. In particular, the homology class of any algebraic subvariety of a complex torus in a suitable (i.e. sufficiently rich) toric compactification is known as the tropicalization of the subvariety, so the problem is to compute tropicalizations of multisingularity strata of the map F.

I will explain the solution for the simplest stratum (i.e. for the bifurcation set of F): the bifurcation set B is always a hypersurface, which resembles the results of Jelonek on the non-properness set. The tropicalization of B (i.e. the Newton polytope of its equation) can be computed in the spirit of Gelfand-Kapranov-Zelevinsky work on Newton polytopes of discriminants. These results are based on a certain characterization of Newton-nondegenerate systems of equations in the spirit of tameness. I will also outline the solution for the second simplest stratum, which is the stratum of double points.

HOMOTOPY CLASSIFICATION OF MONOLITERAL PHRASES AND ITS APPLICATION TO CURVES ON SURFACES

FUKUNAGA TOMONORI

The study of curves via words was introduced by C. F. Gauss. Gauss encoded closed planar curves by words of certain type which are now called Gauss words. We can apply this method to encode multi-component curves on surfaces.
V. Turaev studied stable equivalence classes of curves and links on surfaces by using generalized Gauss phrases (called nanophrases). Turaev introduced the homotopy equivalence on the set of nanophrases and he showed that a stable equivalence class of an oriented, ordered, pointed multi-component curve (respectively link diagram) on a surface is identified with a homotopy class of a nanophrase in a 2-letter (respectively 4-letter) alphabet.
In this talk, we introduce the classification of monoliteral phrases with four or less letters. Furthermore, as a corollary or this result, we give the classification of multi-component curves on surfaces up to stably equivalence with some condition.

Lê-Greuel type formula for the Euler obstruction of a function and applications

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Joint work with: Nicolas Dutertre - Université de Provence, Marseille- France.

1. Abstract

Let $(V, 0) \subset (\mathbb{C}^n, 0)$ be the germ of a reduced equidimensional complex analytic set, and let $f : (V, 0) \to (\mathbb{C}, 0)$ be the germ of an analytic function with isolated singularity.

The Euler obstruction of f, defined in [1], can be looked as a generalization of the Milnor number for functions defined on singular spaces. In this work, using the Euler obstruction of a function, we give a a version of the Lê-Greuel formula for germs $f : (V, 0) \to (\mathbb{C}, 0)$ and $g : (V, 0) \to (\mathbb{C}, 0)$ of analytic functions with isolated singularity at the origin.

Using this formula, we also present a integral formula for the Euler obstruction of a function, generalizing the formula of Kennedy [2].

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Equisingularity and The Euler Characteristic of a Milnor Fibre

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March 30, 2011

The Euler characteristic is the most basic and, possibly, most powerful invariant in topology. It is well known to be crude but extremely effective. A powerful invariant in the study of complex analytic hypersurface singularities is the Milnor fibre. The famous Briançon-Speder-Teissier result states that a family of isolated hypersurface singularities is equisingular if and only if its μ^*-sequence arising from Milnor fibres of slices is constant.

In this talk I will show that if a similar sequence for a family of corank 1 complex analytic mappings from n-space to $n+1$-space is constant, then the image of the family of mappings is equisingular. For families of corank 1 maps from 3-space to 4-space the converse is shown.

These results on equisingularity follow from careful study of the Milnor fibre of non-isolated singularities. The key invariant studied is given in terms of the Euler characteristic of a fibre in between the original singularity and its Milnor fibre and in terms of the Euler characteristics associated to strata of the in-between fibre.

POSTER

CENTROAFFINE HOMOGENEOUS CURVES AND THEIR RELATIONS WITH CENTROAFFINE HOMOGENEOUS SURFACES IN $\mathbb{R}^3$

NA HU

Abstract. I want to talk about the classification the centroaffine space curves with constant centroaffine curvature and constant centroaffine torsion, which can be proved to be the centroaffine homogeneous curves in $\mathbb{R}^3$. As it is known, the homogeneous curves which are precisely the orbits of certain one parameter subgroups G of $GL(3,\mathbb{R})$, and the homogeneous surfaces are the orbits of certain two parameter subgroups $\tilde{G}$ of $GL(3,\mathbb{R})$, considering the relations between groups G of the centroaffine homogeneous curves and the ones $\tilde{G}$ of the centroaffine homogeneous surfaces in $\mathbb{R}^3$, I will finally show G are the subgroups of $\tilde{G}$.

Date: March 31, 2011.

Singularities of Constrained Hamiltonian Systems and Applications in Singular Lagrangian Dynamics

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29 March 2011

We study here the local theory of singularities of pairs (α, ω) consisting of a closed 1-form α and a closed 2-form ω on a manifold M. Any such pair defines a Constrained (or Generalised) Hamiltonian System (CHS) on M , i.e. a differential system of the form

$$i_X\omega = \alpha,$$

by taking the interior product with ω, the (generalised) Hamiltonian being some function f such that, locally, $\alpha = df$. It is known, that in the case where ω degenerates along some points $\Sigma(\omega)$ of the manifold M (i.e. it is not symplectic), existence and uniqueness of solutions in general fails and consequently impasse phenomena occur. We determine here the typical singularities of CHS (α, ω) and we study the geometric properties of the strata of low codimension. Of special interest are the semi-stable cases, i.e. where the 2-form ω has Martinet Σ_{20} or Martinet-Roussarie Σ_{220} stable singularities and α is a closed 1-form, generic relative to the symmetries of ω. This also leads to a complete classification of typical singularities of CHS on 2 and 4-dimensional manifolds. We then give some preliminary normal forms of pairs (α, ω) in order to prove several adjacencies of singularity classes. These preliminary normal forms may also be used in order to determine the local behaviour of the phase curves of a CHS in a neighborhood of an impass point on $\Sigma(\omega)$.

Finally, we apply the results to a classical problem derived from physics, concerning the dynamics of singular Lagrangians, already studied by Dirac, Faddeev-Jackiw and others, mostly for the purposes of the quantisation problem in gauge theories. These Lagrangians are of first order in the velocities, represented locally as

$$L = \beta - f,$$

where β is a (non-closed) 1-form and f is a function on a manifold M and they may be obtained as quantisation limits of regular Lagrangians

$$L = g_m + \beta - f,$$

of the mass m in the kinetic term g_m (the riemannian metric) tending to zero. The singular variational problem for such Lagrangians gives rise to Euler-Lagrange equations which define a CHS on M:

$$i_X d\beta = df.$$

The extremals determine the motion of a charged particle in the limit of a strong magnetic field $d\beta$, in the presence of an external force df. For example, in the 2-dimensional case, for a magnetic field that vanishes non-degenerately along a line $\Sigma(d\beta)$ and in the presence of a generic external force df, we obtain that the particle cannot pass from one side of the plane to the other following some extremal curve, crossing the zero locus. An open and dense subset of the zero locus $\Sigma(d\beta)$ of the magnetic field will consist of either stable, or unstable equilibrium points, while its complement will consist of isolated points on $\Sigma(d\beta)$ from which the particle may pass tangentially (with first order tangency) along an extremal curve. We also obtain similar results for the higher dimensional cases.

POSTER

Title: The Quintics in two variables
Author: León Kushner

Abstract: The object of this talk is the classification of the quintics in two variables. We start with the algebraic models together with their stabilizers. We continue with the topological part studing families which always give product of spheres. Using Maple 12, it is possible to calculate their rank. The work is still unifinished due to the calculation of all cases.

POSTER

Graphs of stable maps of surfaces in the projective plane

Catarina Mendes de Jesus (cmendes@ufv)
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Abstract:

Let $f : M \to N$ be a stable map, where M is a closed surface. According to Whitney, the singular set of f is made of a set of simple curves (without intersections), closed and disjoint in M. These curves separate the surface M into connected components. Meanwhile, the image of the singular set (apparent contour) is a set of curves in N with a finite number of transverse intersections and isolated cusp points. We can associate to these stable maps certain graphs that codify the topological type of their singular and regular sets. In this work we determine which are the graphs that can be associated to stable maps from (non necessarily orientable) surfaces into the projective plane and into the sphere. We also find a necessary and sufficient condition that tells us when some graph can be the graph of a fold map.

This is a joint work with D. Hacon and M. C. Romero Fuster.

POSTER

LÊ NUMBERS OF SEMI-WEIGHTED HOMOGENEOUS ARRANGEMENTS

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Abstract

In this work we study germs of polynomials formed by the product of semi-weighted homogeneous polynomials of the same type, which we call semi-weighted homogeneous arrangements. First we show how to compute the Lê numbers of such polynomials using only the weights, the degree of homogeneity and the number of variables of the germ. We highlight the fact that the computation of the Lê numbers are not always simple, since it can be difficult to find a suitable coordinate system. Moreover, another key point for arriving at our main theorem is discover the number called *polar ratio* of this polynomials class.

As a natural consequence, we show the constancy of the Lê numbers in families of type $F_t = (F_1)_1^{m_1} \cdots (F_t)_l^{m_l}$, where $(F_t)_i = f_i + t\theta_i$, f_i is weighted homogeneous of type $(r_0, \cdots, r_n; D)$ with isolated singularity and the monomials of θ_i have weighted order greater than D. Using the results of David Massey applied to families of function germs, we obtain the constancy of the Milnor fibre in this family of semi-weighted homogeneous arrangements.

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Small growth vector and other invariants in the monster tower for jets of functions $\mathbb{R} \to \mathbb{R}$

Abstract:

Cartan distributions living on the jet spaces $J^k(1,1)$, $k \in \mathbb{N}$, are Goursat, and are homogeneous – feature *no* singularities. The talk will use the tower of manifolds $\mathbb{P}^k(\mathbb{R}^2)$ bigger than $J^k(1,1)$, $k = 0, 1, 2, \ldots$, but of the same respective dimensions as $J^k(1,1)$, with Goursat distributions Δ^k living on them and featuring *all* possible singularities, known to exist since the work [1]. That is, being locally universal within all Goursat distributions in any given corank k. Such a tower is often called Goursat Monster (GM).

Each stage $\mathbb{P}^k(\mathbb{R}^2)$ of GM has been stratified, by Jean [2], into regions, or (G,S,T)-classes, called also (R,V,T)-classes. The small growth vector of Δ^k depends only on the region in question, and can be effectively expressed in terms of region's (G,S,T) word, by [4] and [5]. One of our objectives is to present and explain this function {regions} $\to$ {small growth vectors}.

Regions, or (R,V,T)-classes possess also another important invariant, emanating from the work [3] – the Puiseux characteristic [of certain family of plane curves related to a given region in any given stage of GM]. Recently a student of Montgomery is being directly relating, [6], the Puiseux characteristic of an (R,V,T)-class to the small growth vector of that class, progressing on one of the open questions in [3]. To elucidate this relation is another objective of the talk.

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Topological classification of finitely determined map germs from $\mathbb{R}^3$ to $\mathbb{R}^3$

Juan Antonio Moya Pérez
(joint work with Juan José Nuño Ballesteros)
Universitat de València

The link of a real analytic map germ $f : (\mathbb{R}^3, 0) \to (\mathbb{R}^3, 0)$ is obtained by taking the intersection of the image with a small enough sphere S^2_ϵ centered at the origin in $\mathbb{R}^3$. If f is finitely determined, then we can ensure that the link is a stable map γ from S^2 to S^2. We define Gauss words which contain all the topological information of the link in the case that the singular set $S(\gamma)$ is connected, and we prove that in this particular case they are a complete topological invariant. As an application, we give a classification of finitely determined weighted homogeneous map germs with 2-jet equivalent to (x, y, xz) in a special case.

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POSTER

Families of curve congruences on Lorentzian surfaces

Ana Claudia Nabarro (USP - Brazil)
Joint work with Amani Saloom

In [A.C. Nabarro and F. Tari, Families of curve congruences on Lorentzian surfaces and pencils of quadratic forms. Proc. Roy. Soc. Edinburgh Sect. 141A, 2011], we define and study the families of conjugate and reflected curve congruences associated to a self-adjoint operator A on a smooth and oriented surface M endowed with a Lorentzian metric. These families parametrise parts of the pencils of forms that link, respectively, the equation of the A-asymptotic curves and that of the A-principal curves, and the equation of the A-characteristic curves and that of the A-principal curves. (There is a crucial difference with the Riemmanian case due to the existence of light-like curves.) In this joint work with A. Saloom, we study the generic local singularities in the members of the families of conjugate and reflected curve congruences and describe how these bifurcate within the families.

THOM POLYNOMIAL ASSOCIATED TO THE MILNOR NUMBER OF ISOLATED COMPLETE INTERSECTION SINGULARITIES

TORU OHMOTO

Abstract. We discuss an equivariant theory of the Chern-Schwartz-MacPherson class for singular varieties with group action and its application to Thom polynomial theory for singularities of maps.

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POSTER

Abstract (poster)

The Milnor number of a determinantal variety germ and of a function germ on it

Nuño-Ballesteros, J.J.; Oréfice, B.; Tomazella, J.N.

Given $F : (\mathbb{C}^N, 0) \to M_{m,n}(\mathbb{C})$ a holomorphic function germ, let $(X, 0)$ be the isolated determinantal singularity given by $X = F^{-1}(M^s_{m,n}(\mathbb{C}))$ where $M^s_{m,n}(\mathbb{C})$ is the set of the complex matrices with rank less then s, with s an integer number between 0 and $\min\{m, n\}$ such that $N < (m - s + 2)(n - s + 2)$, we will define the Milnor number of $(X, 0)$ and the Milnor number of a holomorphic function germ with an isolated singularity at X, $f : (X, 0) \to \mathbb{C}$.

MORSE INDEX OF A CYCLIC POLYGON

GAIANE PANINA

This is a joint work with Alena Zhukova.

It is proven by G. Khimshiashvili and the speaker that cyclic configurations of a planar polygonal linkage are critical points of the signed area function. In the talk we give an explicit formula of the Morse index for the signed area of a cyclic configuration. We show that it depends not only on the combinatorics of a cyclic configuration, but also on its metric properties.

Graham Reeve (Liverpool)
Singularities of the affine chord envelope (centre symmetry set) for two surfaces in 4-space

I shall discuss Minkowski sets (Centre Symmetry Sets) for a pair of surfaces in $\mathbb{R}^4$. Given two 2-surfaces M and N in 4-dimensional affine space we consider pairs of points $a \in M$ and $b \in N$ such that the tangent planes T_aM and T_bN are not in general position. We say that two 2-planes in $\mathbb{R}^4$ are in *general position* if their parallels span $\mathbb{R}^4$ as a linear space $T_aM \bigoplus T_bN = \mathbb{R}^4$. If $T_aM \bigoplus T_bN \neq \mathbb{R}^4$ the pair of points a, b is called *special*. A straight line through a special pair is called a *chord*. The envelope of these chords for two generic 2-surfaces is called the *Minkowski set* and I shall classify its generic local singularities. These singularities can be either V. Arnold's A_k type for $k = 2, ..., 5$, V. Arnold's standard boundary singularities of the types B_k or C_k for $k = 2, ..., 5$, F_4, one of the non-simple singularties of the type $F_{1,0}$ of $K_{4,2}$ [1] or one of V. Goryunov's bifurcation diagrams of functions on complete intersections of types $C_{2,2}$ and $C_{3,2}$ [3].

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POSTER

NUMBER OF SINGULARITIES IN QUASI HOMOGENEOUS MAP GERMS FROM $(\mathbb{C}^{n+3}, 0)$ TO $(\mathbb{C}^3, 0)$

E. C. RIZZIOLLI, A. J. MIRANDA, AND M.J.SAIA

The study of the 0-stable types that appear in a generic deformation of a finitely determined map germ $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^p, 0)$ is very relevant because they hide information about the geometrical and topological behavior of map-germs. In particular, if the map germ is weighted homogeneous there exists the interest to describe the number of these singularities through the weights and degrees. Gaffney and Mond obtained formulae to compute the number of cusps and double-folds of map germs from the plane to the plane that involve only the weights and degree. In the case of germs of surfaces in the space Mond also derives formulae for number of the triple points and cross-caps in terms of the weights and degree of the map germ; and when the source dimension and target dimension coincide Marar, Montaldi and Ruas in show the famous expressions to compute all 0-stable singularities in the co-rank one case.

Here we describe how to compute all 0-stable singularities that appear in the discriminant of a stable deformation of weighted homogeneous finitely determined map germs from $(\mathbb{C}^{n+1}, 0)$ to $(\mathbb{C}^n, 0)$ with $n = 2$ or 3 also in the co-rank one case. The main motivation for this work is of course when $n = 3$ and in this case, comparing with the case that the dimension of the source and of the target are equal, the difficulty increases a lot. When the source and the target are equal, all formulae shown by Marar, Montaldi and Ruas are direct consequence of the Greuel-Hamm Theorem for algebras defined by ideals which are associated to Isolated Singularity Complete Intersections (ICIS). When the dimension of the source is greater than the dimension of the target this is not true and we need to apply other methods to compute these numbers.

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Singularities of Abel–Jacobi maps and geometry of dissolving multivortices

Nuno M. Romão
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Gauged vortices are configurations of fields for certain gauge theories on fibre bundles over a complex curve Σ. Their moduli spaces support natural L^2-metrics, which are Kähler, and whose geodesic flow approximates vortex scattering at low speed. In the simplest example of line bundles, the moduli spaces are modelled on the spaces $\Sigma^{(d)}$ of effective divisors on Σ with a fixed degree d.

In my talk, I shall describe the behaviour of these L^2-metrics when the vortices approach a "dissolving limit" in which the L^2-geometry simplifies. In the generic situation of this limit, the metrics degenerate precisely at the singular locus of the Abel–Jacobi map AJ at degree d, and I will explain how to think of this in terms of the geometry of the singular variety $AJ(\Sigma^{(d)})$ inside the Jacobian of Σ. I will also discuss the geodesic flow close to a singularity in the simplest example, and how it relates to the problem of locating Weierstrass points on Σ.

Topological classification of co-rank two map germs from the plane to the plane.

M. J. Saia-ICMC,USP, São Carlos, SP, Brazil

Joint work with A. J. Miranda-UNIFAL and L. M. F. Soares-UFPi,

Abstract

We study the topological classes in $\mathcal{K}$-orbits of corank two finitely determined map germs from the plane to the plane. According to Gaffney and Mond these $\mathcal{K}$-orbits have normal forms given by a quasi homogeneous map germ of type $(xy, x^a + y^b)$, with $(a, b) = 1$. To study these orbits we investigate the topological invariants (the number of vanishing cusps and double folds) which appear in germs of an orbit. First we show how to obtain the number of vanishing cusps in terms of Newton non degeneracy conditions associated to the defining equation of the critical set of the germ. To study the double folds we use the relations among them and the first Fitting ideal of the discriminant curve. We show formulae to determine these numbers and show the topological orbits of a $\mathcal{K}$-orbit $(xy, x^a + y^b)$ for a very large number of pairs (a, b) with $(a, b) = 1$.

On the topology of cooriented wave fronts in spaces of small dimensions

Vyacheslav Sedykh

Moscow, Russia

We consider Legendre singularities with respect to Legendre equivalence preserving a coorientation of the contact structure. In this case, we calculate the adjacency indices of multisingularities of generic Legendre mappings to smooth manifolds of the dimension $n \leq 6$. As a corollary, we find new coexistence conditions on singularities of wave fronts. Namely, we find all linear relations with real coefficients between the Euler characteristics of manifolds of singularities of any generic compact cooriented wave front in any n-dimensional space.

POSTER

Divergence of the reduction to the multidimensional Takens normal form

Ewa Stróżyna, Henryk Żołądek

May, 2011

Abstract

In our previous work a generalization of the Takens normal form for a nilpotent singularity of a vector field in $\mathbb{C}^n$ was obtained. Here we present an example where the corresponding normalizing series is divergent. This indicates that the generalized Takens normal form for a general nilpotent singularity in $\mathbb{C}^n$, $n \geq 3$, is non-analytic.

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Generic classifications of first order multi-Clairaut type equations

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Abstruct

Clairaut type equations are the typical examples of implicit first order ordinary differential equations with classical complete solutions, like as the Clairaut equations. A characterization and a generic classification of such equations have been given by S. Izumiya and Y. Kurokawa. The main result in this talk is to give a generic classification of semi-local cases of Clairaut type equations (namely, multi-Clairaut type equations). The multi-Clairaut type equations are related to implicit systems of first order differential equations on the plane and divergence diagrams.

POSTER

LAGRANGIAN TANGENCY ORDERS AND THE INDEX OF ISOTROPY

ŻANETA TRĘBSKA

Abstract. We study the local symplectic algebra of curves. Following ideas from [2] and [5] we define discrete symplectic invariants - the Lagrangian tangency orders and compare them with the index of isotropy. Although this invariant has definition similar to the index of isotropy its nature is different. Since the Lagrangian tangency order takes into account the weights of quasi-homogeneity of curves it allows us to distinguish more symplectic classes in many cases. For example using the Lagrangian tangency order we are able to distinguish classes E_6^3 and $E_6^{4,\pm}$ of classical planar singularity E_6 which can not be distinguished nor by the isotropy index nor by the symplectic multiplicity. On the other hand, there are singularities which symplectic classes can be distinguished by the index of isotropy but not by the Lagrangian tangency order, for example the parametric curve with semigroup $(3, 7, 11)$. These examples show that there are no simple relations between the Lagrangian tangency order and the index of isotropy even for the case of parametric curves.

This is a joint work with Wojciech Domitrz.

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Geometry of tangent sections of smooth surfaces
Ricardo Uribe-Vargas (U de Bourgogne)

Joint work with Peter Giblin

We consider the following question: given a smooth surface and a plane intersecting it along a curve, we are interested in the quantity of vertices and inflections of that curve and how they are arranged when that plane is very close to be tangent.

If we take an ellipsoid, that plane curve will be a small ellipse, which has, as is known, four vertices. It happens that if the surface is locally a small deformation of an ellipsoid, then the plane section will have again four vertices, but if the surface looks locally as a one-sheet hyperboloid (a saddle), then there are various qualitatively different cases (for the quantity and arrangements of vertices and inflections).

In the talk, it will be shown how to obtain the arrangement and quantity of vertices and inflections of the section of such a surface with a plane close to a tangent one, without bulky calculations.

This question will be also considered from the point of view of the surface as a whole: the domains in which points show the same behaviour are separated by some relevant curves of the surface - the "vertex curve" and the flecnodal curve. The local behaviour of vertices and inflections at the points of these curves will be also described.

GEOMETRY OF POLYNOMIAL MAPPINGS AT INFINITY VIA INTERSECTION HOMOLOGY

ANNA VALETTE

Abstract. We associate to a given polynomial map $F : \mathbb{C}^2 \to \mathbb{C}^2$ with nonvanishing jacobian a variety whose homology or intersection homology describes the geometry of singularities at infinity of this map.
It is joint work with Guillaume Valette.

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$\mathcal{C}^0$ AND BI-LIPSCHITZ $\mathcal{K}$-EQUIVALENCE

GUILLAUME VALETTE

Abstract. I will give several results on the classification of smooth mappings up to $\mathcal{K}$-equivalence (contact equivalence). We will deal with both $\mathcal{C}^0$ and bi-Lipschitz $\mathcal{K}$-equivalence of mappings $F : \mathbb{R}^m \to \mathbb{R}^p$.
It is joint work with Maria Ruas.

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SLANT GEOMETRY OF SPACELIKE HYPERSURFACES IN THE LIGHTCONE WITH RESPECT TO THE ϕ-DE SITTER DUALS

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Abstract

One-parameter families of Legendrian dualities for pseudo-spheres in Lorentz-Minkowski Space were obtained in [13] as extensions of four dualities in [12]. Moreover, new extrinsic differential geometries on spacelike hypersurfaces in these pseudo-spheres were constructed in [3,13,14] as applications of such extensions.

In this talk, a new extrinsic differential geometry on spacelike hypersurfaces in the lightcone is given with respect to the ϕ-de Sitter duals [14].

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POSTER

GEOMETRIC CHARACTERIZATION OF MONGE-AMPÈRE EQUATIONS

ATSUSHI YANO

I'll talk about characterization of Monge-Ampère equation

$$Az_{xx} + 2Bz_{xy} + Cz_{yy} + D + E(z_{xx}z_{yy} - z_{xy}^2) = 0,$$

where each capital letter indicates a function of variables x, y, z, z_x, z_y. A single second order PDE of one unknown function with two independent variables corresponds to a hypersurface R in 2-jet space $J^2(\mathbb{R}^2, \mathbb{R})$ with coordinates (x, y, z, p, q, r, s, t) $(p := z_x, q := z_y, r := z_{xx}, s := z_{xy}, t := z_{yy})$. On the other hand, it is well-known that a Monge-Ampère equation can be expressed in terms of exterior differential system (EDS)—*Monge-Ampère system* $\mathcal{I}$. I studied the relation between *Monge characteristic systems* of Monge-Ampère equation R and *those* of Monge-Ampère system $\mathcal{I}$. From this observation, I characterized Monge-Ampère equations as hypersurfaces R in $J^2(\mathbb{R}^2, \mathbb{R})$.

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POSTER

Wataru Yukuno (Hokkaido), Nilpotent approximation of bracket-generating family of vector fields, and its application

I would like to explain an estimate of a subriemannian ball on a Carnot-Caratheody distance.

In order to estimate the subriemannian ball, I introduce an approximation of vector-field system. In particular, the minus-firstly approximated vector-field system is globally bracket-generating. The Lie algebra genareted by the minus-firstly approximated vector-field system is nilpotent. By using the Carnot-Caratheody distace canonically difined by the minus-first approximated vector-field system, I would like to explain an estimate of a subriemannian ball on a Carnot-Caratheody distance.