Banach Center Conferences

Workshop on Singularities in Geometry and Applications

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Abstracts of invited talks and minicourses

FINAL VERSION

MINICOURSE

Implicit Differential Equations

Alexey Davydov and Farid Tari

In the simplest case, an implicit differential equation (IDE) could be defined as the zero level of a smooth (or analytic) function on the projectivization of the tangent bundle of a two dimensional manifold. In an appropriate local coordinates near any point of this level, the equation takes the form

$$F(x, y, p) = 0,$$

where F is the function and $p = dy/dx$. IDEs appear in various fields and the course will cover examples from control theory and differential geometry.

The first two lectures will be given by Alexey Davydov and will cover the following topics:

- Examples of implicit ODEs and their solutions: characteristic directions in PDEs, limit directions in control systems; implicit two dimensional autonomous systems of first order ODEs;
- General notion of implicit ODEs, regular and singular points; notion of normal forms;
- Traditional proof of Cibrario-Tricomi normal form;
- Topological moduli at pleated point;
- Folded points: reduction theorem and parametric reduction theorem;
- Complete local classification of main symbols of generic linear second order PDEs in the plane;
- Controllability of systems on two-dimensional orientable surfaces and its structural stability;

– Classification of local singularities of implicit two dimensional autonomous systems of first order ODEs.

The remaining two lectures will be given by Farid Tari and here are their abstracts.

Lecture 3: *Pencils of quadratic forms on surfaces*

Binary differential equations are equation written in the form

$$a(x,y)dy^2 + 2b(x,y)dxdy + c(x,y)dx^2 = 0 \tag{1}$$

where a, b, c are smooth functions $U \subset \mathbb{R}^2 \to \mathbb{R}$. BDEs model many pairs of foliations on surfaces in the Euclidean space $\mathbb{R}^3$ such as the lines of principal curvature (P), asymptotic curves (A) and characteristic curves (C). At each point $(x, y) \in U$, the BDE (1) can be considered as a quadratic form and be represented by a point $[a(x, y) : b(x, y) : c(x, y)]$ in the projective plane $\mathbb{R}P^2$. Then the representatives of (A), (P) and (C) in $\mathbb{R}P^2$ are related: they form a self-polar triangle with respect to the cone of degenerate quadratic forms. Working in $\mathbb{R}P^2$ allows the discovery of several natural 1-parameter families of BDEs on surfaces, and these are obtained by considering pencils of quadratic forms in $\mathbb{R}P^2$. The relation between (A), (P) and (C) in $\mathbb{R}P^2$ can also be used to define lines of principal curvature on surfaces in $\mathbb{R}^4$ and to propose a definition of these lines on surfaces $\mathbb{R}^5$ using invariants of binary forms.

Lecture 4: *Singularity theory in differential equations*

By applying the Lengendre transformation to an IDE F, one obtains a new IDE G which has the property that the solution curves of $G = 0$ are dual to those of $F = 0$. We explore this duality at some singularities of IDEs. The talk will also cover some aspects of applications of singularity theory to the study of IDEs. For instance, the singularities of the discriminants of BDEs are best understood when viewing the discriminants as determinants of families of symmetric matrices. This approach proved to be useful when considering bifurcations in families of BDEs. The talk will also cover the use of Legendrian singularities in the study of IDEs (and PDEs) with first integrals.

MINICOURSE

Singularity Theory techniques in Extrinsic Geometry

M.C. Romero Fuster

Singularity theory tools, as illustrated by several papers which appeared during the last decades ([2], [3], [4], [5], [6], [8], [9], [10], [11], [12], [13], [14], [15], [17], [20], [21], [22]), have proven to be useful in the description of geometrical properties of submanifolds immersed in different ambient spaces from both, the local and global viewpoint. The natural connection between Geometry and Singularities relies on the basic fact that the contacts of a submanifold with the models (invariant under the action of a suitable transformation group) of the ambient space can be described by means of the analysis of the singularities of appropriate families of contact functions, or equivalently, of their associated Lagrangian and/or Legendrian maps ([1], [16], [18], [23]).

Our objective is to illustrate how the application of typical techniques in Singularity Theory help in the description of geometrical properties of submanifolds generically immersed in Euclidean space, leading to new results on local and global aspects. We shall mainly concentrate our attention in the case of surfaces immersed in $\mathbb{R}^4$ and $\mathbb{R}^5$.

The course consists in 4 lectures.

Lecture 1: Singularities and contacts.

We discuss the relevance of the concept of contact in geometry and recall some basic definitions and results of Singularity Theory that will be used in the following lectures: $\mathcal{K}$-equivalence, versal deformations and unfoldings, catastrophe maps and bifurcations sets, Thom-Boardman stratification and Boardman symbols.

Lecture 2: Genericity.

We review Montaldi's result on the characterization of contacts through the singularities of contact maps ([16]) and apply it to the study of

the contacts of submanifolds with hyperspheres and hyperplanes in Euclidean space. We also review Montald's genericity theorem ([18]) and analyze, as a consequence, the singularities of the families of distance squared and height functions on submanifolds generically immersed in $\mathbb{R}^n$. We describe how the singularities of these functions are related to the extrinsic geometry of the submanifolds.

Lecture 3: Surfaces in $\mathbb{R}^4$.

We introduce basic geometrical concepts associated to the second fundamental form on surfaces in Euclidean space, such as the shape operator and the curvature ellipse. The study of the curvature ellipse leads to a classification of the points of a surface in 4-space into elliptic, parabolic, hyperbolic, semiumbilic and inflection points as well as to the definition of asymptotic directions. The analysis of the singularities of the height functions and distance squared functions on the surfaces immersed in $\mathbb{R}^4$ allows us to obtain results concerning the generic distribution of these points as well as the generic configurations of the asymptotic lines. We describe global results on existence of semiumbilic, parabolic cusp, flat ridges and inflection points. We also discuss the relation between convexity, semiumbilicity, hypersphericity and the behaviour of the asymptotic lines.

Lecture 4: Surfaces in $\mathbb{R}^5$.

Surfaces in 5-space have a richer second order geometry. We explore the particularities of this case and classify the points of the surfaces according to the behaviour of the cone of contact directions and of the affine span of the curvature ellipse in the normal space. We introduce the concept of asymptotic directions and study the generic behaviour of the foliations of asymptotic lines. We characterize the 2-singular points ([7], [19]) as corank 2 singularities of height functions and study their existence and generic distribution in connection with the asymptotic foliations. We also analyze the singularities of distance squared functions and characterize their corank 2 singularities. This leads to the concept of umbilical curvature function and the characterization of its zeroes as the 2-singular points of the surface.

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Topological $\mathcal{K}$-equivalence of map germs

J.J. Nuño-Ballesteros

(Joint work with J.C. Ferreira Costa)

Abstract

We are interested in the topological classification of map germs $f : (\mathbb{R}^n, 0) \to (\mathbb{R}^p, 0)$. In previous works, we have considered the topological $\mathcal{A}$-classification of $\mathcal{A}$-finitely determined map germs for some values of (n, p) with $n \leq p$. According to a theorem due to Fukuda, f has a cone structure over its *link*, obtained as the intersection of the image of f with a sufficiently small sphere S_ϵ^{p-1} centered at the origin in $\mathbb{R}^p$. By Fukuda's theorem, the inverse image $f^{-1}(S_\epsilon^{p-1})$ of S_ϵ^{p-1} is diffeomorphic to the $(n-1)$-sphere S^{n-1} and the restriction $f : f^{-1}(S_\epsilon^{p-1}) \to S_\epsilon^{p-1}$ is a topologically stable map (C^∞-stable if we are in the nice dimensions). We call this map the link of f. This link is well defined up to stable isotopy and its isotopy class determines the topological $\mathcal{A}$-class of f.

In this talk, we will explain how to adapt this construction in order to study topological $\mathcal{K}$-equivalence of $\mathcal{K}$-finitely determined map germs, including also the case $n > p$. Now the link is not stable anymore, but it is well defined up to homotopy and that its homotopy class determines the topological $\mathcal{K}$-class of f. As a consequence, we will deduce some known results: (1) if $n = p$ then two map germs f, g are topological $\mathcal{K}$-equivalent if and only if $|\deg(f)| = |\deg(g)|$ (Nishimura), or (2) if $n < p$, then two map germs f, g are always topological $\mathcal{K}$-equivalent (Costa). We also discuss about other new results of the same nature.

Some insights on the Euler local obstruction

Jean-Paul Brasselet, CNRS, Marseille France

The lecture concerns joint work with N. Grulha and M. Ruas.

The local Euler obstruction was first introduced by R. MacPherson as a key ingredient for his construction of characteristic classes of singular complex algebraic varieties. Then, an equivalent definition was given by J.-P. Brasselet and M.-H. Schwartz using vector fields. This new viewpoint brought the local Euler obstruction into the framework of "indices of vector fields on singular varieties". There are various other definitions and interpretations in particular due to Gonzalez-Sprinberg, Verdier, Lê-Teissier and others, and there is a very ample literature on this topic.

Then, the notion of local Euler obstruction developed mainly in two directions: the first one comes back to MacPherson's definition and concerns with differential forms. That is developed by W. Ebeling and S. Gusein-Zade in a series of papers. The second one relates local Euler obstruction with functions defined on the variety (J.-P. Brasselet, Lê D. T., D. Massey, A. J. Parameswaran and J. Seade) and with maps (N. Grulha). That approach is useful to relate local Euler obstruction with other indices (paper with N. Grulha and M. Ruas). Aim of the lecture is to present together these new features on the subject.

“Medial/Skeletal Linking Structures and the Geometry of Multi-Object Configurations”

James Damon

Abstract:

In this talk we will introduce an approach for determining geometric properties of a collection of generic objects $\{S_i\}$ in $\mathbf{R}^n$, (i.e. compact regions with piecewise smooth generic boundaries). This includes analyzing both the contributions of both the shapes of the objects and their positional geometry to the geometric properties of the configuration. Such configurations model collections of physiological features in medical images which motivate a number of questions being considered.

We do this by extending the medial/skeletal representations (M_i, U_i) of the individual objects, which consists of a Whitney stratified set M_i in S_i and multi-valued vector field U_i on M_i. This structure and the mathematical objects defined using it already capture the local, relative and global geometry as well as the topology of the individual object S_i.

The linking structure augments this with stratified linking functions l_i and associated linking vector fields L_i which are defined on a refinement on the Whitney stratified sets M_i. These refinements are defined via singularity-theoretic properties of families of “multi-distance functions” on the region boundaries. The properties of the refined stratification result from a transversality theorem for these multi-distance functions.

Applying the mathematical operations to these linking structures on the M_i allow us to compute the geometry in the complements of the objects as well as providing a number of positional invariants for the configuration.

This represents joint work with Ellen Gasparovic.

Singularities of robot manipulators:
Lie groups and exponential products

Peter Donelan

The kinematics of a robot manipulator are described by a relation between its *jointspace*, generally a subvariety of a torus or the product of a torus and a Euclidean space, and its *workspace*, usually a subset of the Lie group of Euclidean motions $SE(3)$. Depending on the architecture of the manipulator, either the forward or the inverse kinematics may be a function which, for topological reasons, must have singularities. For the robot engineer, it is essential to know about these as they lead to a variety of phenomena, such as loss of control, excessive joint acceleration or torque and trajectory branching. There is an extensive engineering literature on robot singularities and a much smaller literature applying singularity-theoretic techniques to understand them.

I will give some background to the problems from both engineering and mathematical points of view, then describe some recent work on manipulators whose links are joined in series. Here, the forward kinematics is most succinctly described by a product of exponentials in the Lie group. The special form of the kinematic mappings leads to conditions for singularities and genericity in terms of the group's Lie algebra.

Stable unfoldings of map-germs on singular varieties

Let $V \subset \mathbf{C}^n$ be open, let $X \subset V$ be an analytic variety, let $S \subset X$ be a finite set, and let $f : (X, S) \to (\mathbf{C}^p, 0)$ be a germ of analytic map.

I will sketch how to construct topologically stable unfoldings of such germs under very mild conditions, conditions so mild that the germs for which they *do not* hold form a subset of infinite codimension (in the sense pioneered by Tougeron).

There are several ingredients:

(1) Critical sets for analytic maps on singular varieties via stratifications
(2) Stratifications and fine resolutions (with due deference to Hironaka)
(3) Jacobian ideal-sheaves in the contexts of (1) and (2).
(4) Variations on the Thom-Mather theory that produces smooth equivalences out of commutative algebra.
(5) Critical value stratifications (with due deference to Looijenga).

I will briefly describe these, and how they fit together to prove the result claimed.

Andrew du Plessis

Local invariants of maps between 3-manifolds

Victor Goryunov (Liverpool)

Abstract

We classify order 1 invariants of maps from closed 3-manifolds to $\mathbb{R}^3$ whose increments are defined only by local bifurcations. We show that in the oriented case the space of integer invariants has rank 7, and give a geometric interpretation of its basis. The $\mathbb{Z}_2$ setting adds another 4 linearly independent invariants, one of which combines the self-linking of the cuspidal edge of the critical value set with the number of connected components of the edge. The ranks of the spaces of the integer and mod2 invariants in the case of a non-oriented source turn out to be respectively 4 and 6. The results provide estimates on the ranks of the invariant spaces for an arbitrary target 3-manifold. The proofs are based on the study of bifurcations in generic 1- and 2-parameter families of maps.

Singularities of Tangent Varieties to Curves and Surfaces

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Abstract.

Embedded tangent spaces to a submanifold draw a variety in the ambient space, which is called the *tangent variety* to the submanifold. Tangent varieties appear in various geometric problems and applications naturally. It is well-known, in the three dimensional Euclidean space, that the tangent variety (tangent developable) to a generic space curve has singularities each of which is locally diffeomorphic to the *cuspidal edge* or to the *folded umbrella* (*cuspidal cross cap*). If we consider a curve together with its osculating framings, we are led to the classification of tangent varieties to generic *osculating framed* curves (possibly with singularities in themselves) in the three dimensional projective space. Then the list consists of 4 singularities: the cuspidal edge, the folded umbrella and moreover the swallowtail and the *Mond surface* (*'cuspidal beak to beak'*). In this talk we present the classification results on generic singularities of tangent varieties to curves (resp. osculating framed curves) with arbitrary codimension in projective spaces. Moreover we give the generic diffeomorphism classification of singularities on tangent varieties to *contact-integral* curves (resp. osculating framed contact-integral curves) in a *contact* projective space ($P(V)$, for a symplectic vector space V). The generic classifications of singularities are performed in terms of certain geometric structures and differential systems. Finally we mention the classification problem of tangent varieties to *surfaces*, exhibiting some examples (and some difficulties).

Stabilization of cohomology classes represented by singularity loci

M.Kazarian (MIAN & IUM, Moscow)

Points with the given local singularity types form loci on the source manifold of a holomorphic mapping. In the same way, the multisingularities form loci on the target manifold. The cohomology classes represented by these loci are given by universal polynomials in the Chern classes of the source and the target manifolds. For the local singularities they are known as the Thom polynomials, and for the multisingularities these are the so called residual polynomials. In opposite to the case of Thom polynomials for local singularities, the existence theorem for the universal formula of multisingularities is not proved yet in its full generality.

In the talk, We discuss the stabilization property of the universal polynomials with the growth of the dimensions of the manifolds. In the case of local singularities this stabilization was discovered first by Feher and Rimany. Its geometric reason was explained by Kazarian using the Hilbert schemes. The numerical experiments show that a similar stabilization holds for the residual polynomials of multisingularities. This gives the hope to use the Hilbert scheme approach to complete the proof of the multisingularity formula.

THE GEOMETRY OF FAST AND SLOW DYNAMICS IN NERVE IMPULSE

ISABEL S. LABOURIAU

Abstract. We will discuss the role of different time-scales in the dynamics of models for excitable tissue, like nerve impulse, and the associated geometry. These are important both in analysing existing models and in the construction of new ones.

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Walter Neumann, Columbia University

Local bilipschitz geometry of complex surfaces.

Abstract: I will describe work in progress with Lev Birbrair and Anne Pichon to give a complete bilipshitz classification of the possible local geometry at a point of a normal complex surface. This geometry can be surprisingly complex and carries a lot of algebro-geometric information about the point.

Kentaro Saji,
Geometry of Wavefronts

We give a definition of coherent tangent bundles, which is an intrinsic formulation of wave fronts. The singular curvature can be defined in this setting and we can prove the Gauss-Bonnet type formulas. In our application of coherent tangent bundles for wave fronts, the first fundamental forms and the third fundamental forms are considered as induced metrics of certain homomorphisms between vector bundles. They satisfy the completely same conditions, and so can reverse roles with each other. For a given wave front of a 2-manifold, there are two Gauss-Bonnet formulas. By exchanging the roles of the fundamental forms, we get two new additional Gauss-Bonnet formulas for the third fundamental form. Using these four formulas, we get several results on the topology and geometry of wave fronts.

GEOMETRIC INVARIANTS ON LORENTZIAN SURFACES IMMERSED IN MINKOWSKI $\mathbb{R}^{3,1}$

F. Sánchez-Bringas (in colaboration with P. Bayard)

We study the invariants of a quadratic map $\mathbb{R}^{1,1} \to \mathbb{R}^2$ modulo the action of the groups of isometries and then describe the corresponding quotient set. Moreover, we describe the curvature hyperbola of a quadratic map in terms of its invariants, and for each geometrical type we determine all the corresponding classes of quadratic maps (Classification Theorem). These results are applied to the analysis of the second fundamental form of Lorentzian surfaces immersed in the Minkowski space of dimension 4, $\mathbb{R}^{3,1}$. Some of these invariants can be considered as new invariants of the surface. Certain foliations depending on the extrinsic geometry of the surface are also analyzed.

Zbigniew Szafraniec, Quadratic forms and intersection numbers for polynomial immersions

Let g be an immersion of an n-dimensional manifold into $\mathbf{R}^{2n}$. H.Whitney defined the intersection number $I(g)$ in terms of self-intersections of g. S.Smale has shown that $I(g)$ may be expressed in terms of a corresponding mapping to the Stiefel manifold.

There will be presented methods, based on those concepts, of computing the intersection number of polynomial immersions as signatures of quadratic forms.

Umbilics of surfaces in the Minkowski 3-space

Farid Tari

Abstract

The Carathéodory conjecture states that any smooth closed and convex surface in the Euclidean 3-space has at least two umbilic points. Various attempts were made to prove this conjecture (see [Guilfoyle and Klingenberg, arXiv:0808.0851v1, 2008] for the latest results on the problem using the mean curvature flow on the space of oriented lines in $\mathbb{R}^3$).

We prove that any closed and convex surface in the Minkowski 3-space of class C^3 has at least two umbilic points. For ovaloids, we can even specify the nature of the umbilic points.

The induced metric on a closed surface in the Minkowski 3-space must degenerate at some point on the surface. We denote by the LD (the locus of degeneracy) the set of such points. On the LD, the tangent plane to the surface is lightlike. This means that the "normal vector" to the surface is tangent to the surface. This should complicate matters (we do not have, for instance, a shape operator on the LD). However, the presence of the LD makes the situation easier than in the Euclidean case. We use some elementary topological arguments and Poincaré-Hopf theorem to prove the result. (The preprint can be downloaded form http://maths.dur.ac.uk/ dma0ft/Publications.html)

NEW INVARIANTS FOR COMPLEX MANIFOLDS AND ITS APPLICATION TO COMPLEX PLATEAU PROBLEM

STEPHEN YAU

Abstract. We introduce some new invariants for complex manifolds. These invariants measure in some sense how far away for the complex manifold to have global complex coordinates. For applications, we introduce two new invariants $f^{(1,1)}$ and $g^{(1,1)}$ for isolated surface singularities. We show that $f^{(1,1)} \geq 1$ if the singularity admits a C^*-action. We also prove $f^{(1,1)} = g^{(1,1)} = 1$ for rational double points and cyclic quotient singularities.

Let X be a compact connected strongly pseudoconvex CR manifold of real dimension $2n-1$ in $\mathbb{C}^N$. It has been an interesting question to find an intrinsic smoothness criteria for the complex Plateau problem. For $n \geq 3$ and $N = n+1$, we found a necessary and sufficient condition for the interior regularity of the Harvey–Lawson solution to the complex Plateau problem by means of Kohn–Rossi cohomology groups on X in 1981. For $n = 2$ and $N \geq n+1$, the problem has been open for over 30 years. we introduce a new CR invariant $g^{(1,1)}(X)$ of X. The vanishing of this invariant will give the interior regularity of the Harvey–Lawson solution up to normalization. In case $n = 2$ and $N = 3$, the vanishing of this invariant is enough to give the interior regularity.

Michail Zhitomirskii

Normal Forms in Singularity Theory versus Differential Geometry

I will compare the objectives and the methods in classification problems of singularity theory, where the main word is "generic", with those in local differential geometry, where the main word is "symmetry". I will discuss the possibility of combining the methods. One of examples will be a simple explanation of the famous work by E. Cartan on classification of (2,5) distributions, published 100 years ago.