

Andrzej Makagon
Hampton University
Abolghassem Miamee
Hampton University

Factorization of the spectrum of a PC sequence

A second order stochastic sequence is a sequence of complex random variables with mean zero and finite second moments. Since we are interested only in correlation properties of a stochastic sequence we adopt a slightly more general definition. A *stochastic sequence* is a sequence $(X(n))$ in a complex Hilbert space H indexed by the set of integers Z . The *correlation function* of the sequence $(X(n))$ is the function on Z^2 defined by $R_X(m, n) = (X(m), X(n))$, where (\cdot, \cdot) denotes the inner product in H . If for every $m, n \in Z$, $R_X(m, n) = R_X(m+1, n+1)$, then the sequence is called *stationary*. If T is a positive integer and $R_X(m, n) = R_X(m+T, n+T)$, $m, n \in Z$, then the sequence is called *periodically correlated (PC) with period T* .

Historically, there are two platforms of analysis of stochastic sequences: *time domain* and *spectral domain* analysis. Time domain analysis deals with studying properties of a sequence through its geometry, while spectral domain analysis is the study of properties of a sequence through properties of its spectrum.

The main idea of the spectral analysis is to represent the sequence as a treatable family of functions in some function space related to the spectrum of a sequence. The spectrum is an “object” (Schwartz distribution, function, or a measure) on $[0, 2\pi)^2$ whose Fourier transform is $R_X(m, -n)$. If this “object” is a measure, i.e. if

$$R_X(m, n) = \int_0^{2\pi} \int_0^{2\pi} e^{-i(mu-nv)} \Gamma(du, dv), \quad m, n \in Z,$$

then the sequence is called *strongly harmonizable*. Both stationary and periodically correlated sequences are strongly harmonizable.

The case of a stationary sequence is especially pleasant because its spectral measure Γ is supported on the diagonal of the square $[0, 2\pi)^2$, so it can be identified with one measure on $[0, 2\pi)$, namely $\gamma_0(\Delta) = \Gamma(\Delta \times \Delta)$. It is easy to see that if a function h and a measure μ are such that

$\gamma_0(du) = |h(u)|^2 \mu(du)$ then the sequence $x(n) = e^{-in\cdot} h(\cdot)$, $n \in \mathbb{Z}$, of functions in $L^2([0, 2\pi), \mu; C)$ has the same correlation as the sequence $(X(n))$. This representation opens doors to huge variety of analytic tools and forms a basis for spectral analysis of stationary sequences.

The spectrum of a PC sequence sits on the union $\bigcup_{j=0}^{T-1} L_j$ of T lines

$$L_j = \{(u, v) \in [0, 2\pi)^2 : v = u + 2\pi j/T \text{ modulo } 2\pi\}, \quad j = 0, \dots, T-1,$$

so it can be identified with the set of T measures, γ_j , $j = 0, \dots, T-1$. The main purpose of this talk is to propose a certain functional representation of a periodically correlated sequence analogous to the sequence $x(n)$ described above for a stationary sequence. This will be proceeded by constructing a simultaneous factorization of all the measures comprising the spectrum of the PC sequence.

Bibliografia

- [1] Makagon, A., Miamee, A., (2012) Spectral Representation of Periodically Correlated Sequences, preprint