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Almost sure local limit theorem for Markov chains

Let $\{\xi_k\}$ be $0-1$ state Markov chain generated by 2×2 matrix $\mathbf{P}=(p_{ij})$, where $p_{ij}>0$, $i,j=1,2$. Set

$$\gamma = 1-p_{12}-p_{21}, \quad \pi_0=\frac{p_{21}}{p_{12}+p_{21}}, \quad \pi_1=\frac{p_{12}}{p_{12}+p_{21}}, \quad \sigma^2=\pi_0\pi_1\frac{1+\gamma}{1-\gamma}.$$

Consider $S_n=\sum_{k=1}^n f(\xi_k)$, where $f(0)=-\pi_1$, $f(1)=\pi_0$. Then

$$\frac{\sqrt{2\pi}}{\ln n}\sum_{\nu=1}^{n}\frac{\sigma}{\sqrt{\nu}}I_{[S_\nu=\kappa_\nu]}\overset{\text{a.s.}}{\to} e^{\frac{-\kappa^2}{2}} \qquad \text{as} \qquad \frac{\kappa_\nu}{\sigma\sqrt{\nu}}\to_\nu \kappa.$$

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