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Stochastic Evolution Equation with Nonlinear Potential

This talk considers the stochastic partial differential equation

$$u_t = \frac{1}{2} u_{xx} + u^\gamma \xi$$

where $\gamma > 1$ and non negative initial condition u_0 satisfying $\int_{\mathbb{S}^1} u_0(x)^{2\gamma} dx < +\infty$. The *space* variable is $\mathbb{S}^1 = [0,1]$, the unit circle, with the identification $0 = 1$ and ξ is a space / time white noise Gaussian random field. The stochastic integral is defined in the sense of Walsh [4](1986). It is shown that there exists a unique solution u such that for all $\alpha < 1$

$$\mathbb{E}\left[\left(\int_0^\infty \int_{\mathbf{S}^1} u(t,x)^{2\gamma} dx dt\right)^{\alpha/2}\right] \leq C(\alpha) < +\infty.$$

The solution also satisfies $\mathbb{E}\left[\int_0^T \left(\int_{\mathbf{S}^1} |u(t,x)|^p dx\right)^{\alpha/p} dt\right] < +\infty$ for all $T < +\infty$, $p < +\infty$, $\alpha \in (0, \alpha_0)$ for some $\alpha_0 < 1$ that does not depend on p.

The problem is of interest following work by Carl Mueller [1](1991), showing existence of a solution which remains bounded for $\gamma < \frac{3}{2}$, [2](1993) showing that the L^∞ norm is infinite in finite time with positive probability for γ sufficiently large and [3](2000) showing that, with positive probability, there is explosion of the L^∞ norm in finite time for $\gamma > \frac{3}{2}$.

The work presented here shows that while the L^∞ norms explode, the L^p norms remain well defined.

Bibliografia

[1] Mueller, C. (1991)*Long Time Existence for the Heat Equation with a Noise Term* Probab. Theory Relat. Fields vol. 90 pp. 505 - 517
[2] Mueller, C.; Sowers, R. (1993)*Blowup for the heat equation with a noise term* Probab. Theory Relat. Fields vol. 97 pp. 287 - 320
[3] Mueller, C. (2000) *The Critical Parameter for the Heat Equation with a Noise Term to Blow up in Finite Time* The Annals of Probability, Vol. 28, No. 4 pp. 1735-1746
[4] Walsh, J.B. (1986)*An Introduction to Stochastic Partial Differential Equations* Lecture Notes in Mathematics, vol. 1180, Springer