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Mocne mieszanie; s-samorozkładalność miar; operatorowa samorozkładalność miar

Wykład Dedykowany Pamięci Kazimierza Urbanika (1930–2005)

We will present two recent results, in the limiting distribution theory, where *the standard stochastic independence* (Mark Kac called this *probability multiplication rule*) is replaced by *the strong mixing condition*.

The s-selfdecomposability notion is related to the *non-linear* shrinking mappings $(U_r, r > 0)$, proposed by K. Urbanik around 1971.

For non-negative stochastically independent variables (X_n) weak limits of sequences

$$U_{r_n}(X_1) + U_{r_n}(X_2) + U_{r_n}(X_3) + \ldots + U_{r_n}(X_n) + b_n \tag{1}$$

were described in Jurek (1972), while the case of Hilbert space valued variables was completely solved in Jurek (1977/81). In Bradley and Jurek (2015) Gaussian limits in (1) were investigated for strong mixing conditions of (X_n). [There are non-Gaussian limits in (1) as well.]

In 60's and 70's of the previous century, it was quite commonly accepted that when one has $\mathbb{R}^d$-valued (or a Banach space valued or a topological group G valued) random vectors (X_n) then the corresponding partial sums should be normalized by

$$A_1(X_1) + A_2(X_2) + \ldots + A_n(X_n) + b_n = A_n \sum_{k=1}^{n} X_k + b_n \tag{2}$$

In the fundamental papers by Urbanik (1972) (the $\mathbb{R}^d$ case) and Urbanik (1978) (the Banach space case) limits of (2) were completely characterized in terms of the characteristic functions. Moreover, his proof was given via Choquet's Theorem on extreme points in a compact convex set. More importantly, with each random vector Z (or a probability measure μ) Urbanik associated the following semigroup of operators:

$$\mathbb{D}(Z) := \{A : Z \overset{d}{=} AZ + Y\} \qquad (\mathbb{D}(\mu) := \{A : \mu = A\mu * \nu\}), \tag{3}$$

where A is a linear operator and Y is stochastically independent of Z (ν is the probability distribution of Y). In the monograph Jurek and Mason (1993), semigroups $\mathbb{D}(X)$ ($\mathbb{D}(\mu)$) were named as *the Urbanik decomposability semigroups* of X (or μ).

The key characterization of limits in (2) is the following:

$$Z \text{ is a limit in (2) iff } \exists\, Q \text{ such that } \{e^{-tQ}, t \geq 0\} \subset \mathbb{D}(Z). \tag{4}$$

In Bradley and Jurek (2016) for strongly mixing sequences (X_n) weak limits in (2) were characterized by proving that the corresponding Urbanik decomposability semigroup $\mathbb{D}(Z)$ indeed contains one-parameter semigroups $\{e^{-tQ}, t \geq 0\}$.

Note. In the schemes (1) and (2) we assume that the corresponding triangular arrays are infinitesimal and the limiting distribution (variables) are full, that is, their supports are NOT contained in any proper hyperplane (shifted proper subspace).

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