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Factorizing multilinear operators on Banach spaces, C^*-algebras and JB^*-triples

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Abstract. In recent papers, the Right and the Strong* topologies have been introduced and studied on general Banach spaces. We characterize different types of continuity for multilinear operators (joint, uniform, etc.) with respect to the above topologies. We also study the relations between them. Finally, in the last section, we relate the joint Strong*-to-norm continuity of a multilinear operator T defined on C^*-algebras (respectively, JB^*-triples) to C^*-summability (respectively, JB^*-triple-summability).

1. Introduction and some known results. In [24], [25] and [22] the Right and the Strong* topologies have been introduced and studied on general Banach spaces. These topologies can be defined in the following way: For each bounded linear operator S between two Banach spaces X and Y, the symbol $|||\cdot|||_S$ will denote the seminorm on X defined by

$$x \mapsto |||x|||_S := \|S(x)\|.$$

The *Strong** ($S^*(X, X^*)$) *topology* on X is the locally convex topology associated to the seminorms $|||\cdot|||_S$ induced by all bounded operators $S : X \to H$, with H any Hilbert space. The *Right topology* on X is the locally convex topology associated to the seminorms $|||\cdot|||_S$ induced by all bounded operators $S : X \to R$, with R any reflexive space [24]. When X is a dual Banach space with predual denoted by X_*, the $S^*(X, X_*)$ *topology* on X is generated by all the seminorms $|||\cdot|||_S$, where S is any weak* continuous linear operator from X into a Hilbert space. It is known that $S^*(X^{**}, X^*)|_X = S^*(X, X^*)$ (see [25]). Note that the Right and the Strong* topologies are particular cases of the topologies defined in [17].

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The main result in [24] establishes that a linear operator $T : X \to Y$ between two Banach spaces is weakly compact if and only if its restriction to the closed unit ball of X is Right-to-norm continuous. For Banach spaces in which the Right and Strong* topologies coincide on bounded sets (for example, C^*-algebras, JB^*-triples, Hilbert spaces), the Strong* topology provides a convenient tool to characterize weakly compact operators (cf. [25]). One of the main results in the paper just cited shows, under some additional hypothesis, that a multilinear operator $T : X_1 \times \cdots \times X_n \to X$ is jointly sequentially Right-to-norm (respectively, Strong*-to-norm) continuous if and only if its Aron–Berner extensions remain X-valued.

In this article we study the multilinear case in more detail. We work with different kinds of continuity (using the above topologies) for a multilinear operator. We study the relations between separate, joint, and uniform Strong*-to-norm (respectively, Right-to-norm) continuity. The last section reveals the connections between joint Strong*-to-norm continuous multilinear operators defined on C^*-algebras (respectively, JB^*-triples) and 2-C^*-summing (respectively, 2-JB^*-triple-summing) multilinear operators. Both notions are closely related to absolutely p-summing and p-dominated multilinear operators.

In the early 1980's A. Pietsch [31] started the study of multilinear summing operators. He introduced the following definitions. Let $X_1, \ldots, X_n$ and X be Banach spaces and $0 < s < \infty$, $1 \leq r_1, \ldots, r_n < \infty$ be such that $1/s \leq 1/r_1 + \cdots + 1/r_n$. A multilinear operator $T : X_1 \times \cdots \times X_n \to X$ is said to be *absolutely* $(s; r_1, \ldots, r_n)$*-summing* if there exists a constant $K \geq 0$ such that for every $k \in \mathbb{N}$ and every $(x_i^j)_{i=1}^k \subset X_j$,

$$\Big(\sum_{i=1}^k \|T(x_i^1, \ldots, x_i^n)\|^s\Big)^{1/s} \leq K \prod_{j=1}^n \sup_{f \in X_j^*,\, \|f\| \leq 1} \Big\{\Big(\sum_{i=1}^k |f(x_i^j)|^{r_j}\Big)^{1/r_j}\Big\}.$$

When $s = p_1 = \cdots = p_n = p$ we say that T is *absolutely* p*-summing*.

In our context, of particular significance is the case when $1 \leq r_1, \ldots, r_n < \infty$ and $1/s = 1/r_1 + \cdots + 1/r_n$. Then, an absolutely $(s; r_1, \ldots, r_n)$-summing multilinear operator $T : X_1 \times \cdots \times X_n \to X$ is called $(r_1, \ldots, r_n)$*-dominated*. We only deal with $(2, \ldots, 2)$-dominated multilinear operators (which we just call 2-*dominated*). By definition, the 2-dominated multilinear operators are those for which there exists a constant K satisfying

$$\Big(\sum_{i=1}^k \|T(x_i^1, \ldots, x_i^n)\|^{2/n}\Big)^{n/2} \leq K \prod_{j=1}^n \sup_{f \in X_j^*,\, \|f\| \leq 1} \Big\{\Big(\sum_{i=1}^k |f(x_i^j)|^2\Big)^{1/2}\Big\}$$

for all $k \in \mathbb{N}$ and $(x_i^j)_{i=1}^k \subset X_j$.

Many contributions have supported the development of the multilinear theory of summing operators (see for example [1], [6], [18], [19], [26], [27] and [28]). The following remarkable definition appeared in [6]: Given $1 \leq$

$p_1, \dots, p_n \le q < \infty$, we say that an n-linear operator $T : X_1 \times \dots \times X_n \to X$ is *multiple* $(q; p_1, \dots, p_n)$*-summing* if there is a constant $K \ge 0$ such that for any $k_1, \dots, k_n \in \mathbb{N}$ and $(x^j_{i_j})_{i_j=1}^{k_j} \subset X_j$, $1 \le j \le n$, we have

$$\Big(\sum_{j=1}^{n}\sum_{i_j=1}^{k_j} \|T(x^1_{i_1}, \dots, x^n_{i_n})\|^q\Big)^{1/q} \le K \prod_{j=1}^{n} \sup_{f \in X_j^*,\, \|f\| \le 1} \Big\{\Big(\sum_{i_j=1}^{k_j} |f(x^j_{i_j})|^{p_j}\Big)^{1/p_j}\Big\}.$$

As above, we will only deal with multiple $(2; 2, \dots, 2)$-summing operators (called *multiple* 2*-summing operators*). It follows from the definitions that every multiple 2-summing operator is absolutely 2-summing, but the converse is false in general.

When the Banach spaces $X_1, \dots, X_n$ enjoy an additional algebraic structure, like C^*-algebras and JB^*-triples, absolutely summing operators belong to the wider classes of C^*-summing and JB^*-triple-summing operators (see [32], [21] and §3 for more details). In Section 3, we shall study 2-C^*-dominated and 2-JB^*-triple-dominated multilinear operators on a product of C^*-algebras and JB^*-triples, respectively. A characterization of joint Strong*-continuity is established in terms of a Grothendieck type inequality as well as in terms of 2-JB^*-triple domination.

In [25, Proposition 3.20] the authors gave a characterization of those multilinear operators on Banach spaces which are jointly $S^*(X_1, X_1^*) \times \dots \times S^*(X_n, X_n^*)$-to-norm (resp. Right$\times\dots\times$Right-to-norm) continuous. This continuity is equivalent to factorizing through n Hilbert spaces (resp. n reflexive spaces). Clearly, a multilinear mapping T is separately Strong* (resp., Right) continuous whenever T is jointly Strong* (resp., Right), while the converse is not always true (cf. [25, Example 3.19]). Note that there is an "overlap", in the case of the Right topology, between [24, 25, 22] and [17, 3, 11, 12]. Indeed, in [12] (see also [13]), starting from the work done in [3] and [11], the authors proved the more complete result below, relating the above kind of continuity to the uniform Right $\times \dots \times$ Right-to-norm continuity on bounded sets. Let $X_1, \dots, X_k$ and X be Banach spaces and let $L^k(X_1, \dots, X_k; X)$ denote the space of all k-linear operators from $X_1 \times \dots \times X_k$ into X; moreover $L(X)$ and Id_X will denote $L(X; X)$ and the identity mapping on X, respectively.

THEOREM 1 ([12, Theorem 4]). *Let $X_1, \dots, X_n$ and X be Banach spaces, and let $T : X_1 \times \dots \times X_n \to X$ be an n-linear operator. The following statements are equivalent*:

(a) *T is jointly Right-to-norm continuous* (*at* 0).
(b) *T factors through the cartesian product of n reflexive Banach spaces.*
(c) *T is uniformly Right $\times \dots \times$ Right-to-norm continuous on bounded sets.*

(d) *For each* $i \in \{1, \ldots, n\}$ *the mapping*

$$T_i : X_i \to L^{n-1}(X_1, \overset{[i]}{\ldots}, X_n; X),$$
$$T_i(x_i)(x_1, \overset{[i]}{\ldots}, x_n) = T(x_1, \ldots, x_n),$$

is (*uniformly*) *Right-to-norm continuous on bounded sets.* ∎

As a consequence of the little Grothendieck inequality for C^*-algebras (cf. [32] and [15]), the Strong* topology of a C^*-algebra A coincides with the so-called *C^*-algebra Strong* topology* of A, that is, the locally convex topology on generated by the seminorms $x \mapsto \phi(x^*x + xx^*)$, where ϕ ranges over the positive functionals in the closed unit ball of A.

Remark 2. It is clear that every Strong*-to-norm continuous operator is automatically Right-to-norm continuous. However, the converse is not always true. Consider for example the operator $T : c_0 \to \ell_8$ defined by $T((x_n)_n) = (n^{-1/4}x_n)_n$. Since ℓ_8 is a reflexive space, T is Right-to-norm continuous. If (e_n) denotes the canonical basis of c_0, then it can be checked that 0 belongs to the Strong* closure in c_0 of the set $\{\sqrt{n}\, e_n : n \in \mathbb{N}\}$ (cf. Exercise 1(a), p. 71 in [33] and the comments preceding this remark). Since for each natural n, $\|T(e_n)\| = \sqrt[4]{n}$, it follows that T is not Strong*-to-norm continuous.

Remark 3. When in Theorem 1 the Right topology and the reflexive spaces are replaced with the Strong* topology and Hilbert spaces, respectively, then (a)⇔(b)⇒(c)⇔(d). However, (c)⇏(a). Indeed, it is known that the Right and $S^*(A, A^*)$ topologies coincide on bounded subsets of every C^*-algebra A (cf. [2, Theorem II.7]). In view of this, Remark 2 provides a counterexample to (c)⇒(a).

The characterization of the uniform $S^*(X_1, X_1^*) \times \cdots \times S^*(X_n, X_n^*)$-to-norm continuity on bounded sets, given below, follows directly from [22, Theorem 2.4], [25, Theorem 2.9] and [12, Proposition 2].

Corollary 4. *Let* $X_1, \ldots, X_n$ *and* X *be Banach spaces, and* T *an element in* $L^n(X_1, \ldots, X_n; X)$. *Then the following statements are equivalent*:

(a) T *is uniformly* $S^*(X_1, X_1^*) \times \cdots \times S^*(X_n, X_n^*)$*-to-norm continuous on bounded sets.*

(b) *For each* $i \in \{1, \ldots, n\}$ *the mapping*

$$T_i : X_i \to L^{n-1}(X_1, \overset{[i]}{\ldots}, X_n; X),$$
$$T_i(x_i)(x_1, \overset{[i]}{\ldots}, x_n) = T(x_1, \ldots, x_n),$$

is (*uniformly*) $S^*(X_i, X_i^*)$*-to-norm continuous on bounded sets.*

(c) *For each* $i \in \{1, \ldots, n\}$, *there exist a bounded linear operator* S_i *from* X_i *into a Hilbert space and a mapping* $N_i : (0, \infty) \to (0, \infty)$ *such* *that*

$$\|T_i(x)\| \le N_i(\varepsilon)\|\|x\|\|_{S_i} + \varepsilon\|x\|,$$

for all $x \in X_i$ *and* $\varepsilon > 0$.

If we assume that $X_1, \ldots, X_n$ *have property* (V) *and for each* i *in* $\{1, \ldots, n\}$, *the Right and the* $S^*(X_i, X_i^*)$ *topologies coincide on bounded subsets of* X_i, *then the previous three statements are also equivalent to the following*:

(d) *For each* $i \in \{1, \ldots, n\}$ *and each* $S^*(X_i, X_i^*)$*-null sequence* (x_k^i) *in* X_i, *we have* $\|T_i(x_k^i)\| \to 0$, *that is*,

$$\limsup_{k\to\infty}\left\{\|T(z_1, \ldots, z_{i-1}, x_k^i, z_{i+1}, \ldots, z_n)\| : \begin{array}{c} z_j \in B_{X_j}, \\ j \in \{1, \overset{[i]}{\ldots}, n\} \end{array}\right\} = 0. \ \blacksquare$$

Statement (c) above guarantees that a multilinear operator T in $L^n(X_1, \ldots, X_n; X)$ is uniformly $S^*(X_1, X_1^*) \times \cdots \times S^*(X_n, X_n^*)$-to-norm continuous on bounded sets if and only if it *almost* factorizes through the cartesian product of n Hilbert spaces.

2. Two more types of continuity. This section begins with a multilinear generalization of [22, Theorem 2.4].

LEMMA 5. *Let* $X_1, \ldots, X_n$ *and* X *be Banach spaces and let*

$$T : X_1 \times \cdots \times X_n \to X$$

be a multilinear operator. Suppose that

$$T|_{B_{X_1}\times\cdots\times B_{X_n}} : B_{X_1} \times \cdots \times B_{X_n} \to X$$

is jointly Strong**-to-norm* (*respectively*, *Right-to-norm*) *continuous. Then there are Hilbert spaces* (*respectively*, *reflexive Banach spaces*) $H_1, \ldots, H_n$ *and bounded linear operators* $S_i : X_i \to H_i$ *such that*

$$\|T(x_1, \ldots, x_n)\| \le \prod_{i=1}^{n}(\|\|x_i\|\|_{S_i} + \|x_i\|)$$

for all $x_i \in X_i$.

Proof. The set

$$\mathcal{O} := \{(x_1, \ldots, x_n) \in B_{X_1} \times \cdots \times B_{X_n} : \|T(x_1, \ldots, x_n)\| \le 1\}$$

is a neighbourhood of 0 in $B_{X_1} \times \cdots \times B_{X_n}$, in the product of the $S^*(X_i, X_i^*)$ topologies. By the definition of the Strong* topology, for each $i = 1, \ldots, n$, there exists a positive constant δ, Hilbert spaces $H_1^i, \ldots, H_{p_i}^i$ and bounded linear operators $S_j^i : X_i \to H_j^i$ $(1 \le j \le p_i)$ such that $\mathcal{O}$ contains the set

$$\mathcal{O}' := \{(x_1, \ldots, x_n) \in B_{X_1} \times \cdots \times B_{X_n} : \|\|x_i\|\|_{S_j^i} \le \delta,\ \forall 1 \le j \le p_i,\ 1 \le i \le n\}.$$

We define

$$H_i := \bigoplus_{1\le j\le p_i}^{\ell_2} H_j^i$$

and let $S_i : X_i \to H_i$ be the bounded linear operator given by $S_i(x_i) := (\delta^{-1} S_j^i(x_i))_j$. Clearly, for each i, H_i is a Hilbert space.

For each $(x_1,\dots,x_n) \in X_1 \times \dots \times X_n$ with $x_i \ne 0$ $(1 \le i \le n)$, the element

$$\left(\frac{1}{\|S_1(x_1)\| + \|x_1\|}\, x_1, \dots, \frac{1}{\|S_n(x_n)\| + \|x_n\|}\, x_n\right)$$

belongs to $\mathcal{O}' \subseteq \mathcal{O}$, and hence

$$\left\|T\left(\frac{1}{\|S_1(x_1)\| + \|x_1\|}\, x_1, \dots, \frac{1}{\|S_n(x_n)\| + \|x_n\|}\, x_n\right)\right\| \le 1,$$

which implies that

$$\|T(x_1,\dots,x_n)\| \le \prod_{i=1}^n (|||x_i|||_{S_i} + \|x_i\|).$$

When $x_i = 0$ for some i, the above inequality is trivial. ■

The next result gives a necessary condition for a multilinear operator to be jointly Strong*-to-norm continuous on bounded sets.

PROPOSITION 6. *Let $X_1,\dots,X_n$ and X be Banach spaces and let*

$$T : X_1 \times \dots \times X_n \to X$$

be a multilinear operator. Suppose that $T|_{B_{X_1}\times\dots\times B_{X_n}}$ is jointly Strong-to-norm (respectively, Right-to-norm) continuous. Then there exist mappings $N_i : (0,\infty) \to (0,\infty)$ (depending only on T), Hilbert spaces (respectively, reflexive Banach spaces) $H_1,\dots,H_n$, and bounded linear operators $S_i : X_i \to H_i$ such that*

$$\|T(x_1,\dots,x_n)\| \le \prod_{i=1}^n (N_i(\varepsilon)|||x_i|||_{S_i} + \varepsilon\|x_i\|) \tag{1}$$

for all x_i in X_i and $\varepsilon > 0$.

Proof. For each natural m, the mapping mT is jointly Strong*-to-norm continuous on $B_{X_1} \times \dots \times B_{X_n}$. Thus, by Lemma 5, there are Hilbert spaces H_m^i $(1 \le i \le n)$ and bounded linear operators $S_{i,m} : X_i \to H_m^i$ such that

$$\|mT(x_1,\dots,x_n)\| \le \prod_{i=1}^n (|||x_i|||_{S_{i,m}} + \|x_i\|)$$

for all $(x_1,\dots,x_n)$. We may assume $S_{i,m} \ne 0$ for all $m \in \mathbb{N}$ and $1 \le i \le n$. Define

$$H_i := \bigoplus_{m\in\mathbb{N}}^{\ell_2} H_m^i,$$

and let $S_i : X_i \to H_i$ be the bounded linear operator given by

$$S_i(x_i) := \left(\frac{1}{m\|S_{i,m}\|}\, S_{i,m}(x_i)\right)_m .$$

Define $N_i : (0,\infty) \to (0,\infty)$ by

$$N_i(\varepsilon) := \frac{m(\varepsilon)}{\sqrt[n]{m(\varepsilon)}}\, \|S_{i,m(\varepsilon)}\|,$$

where

$$m(\varepsilon) = \min\{m \in \mathbb{N} : 1/\sqrt[n]{m} < \varepsilon\}.$$

Finally, given $(x_1,\dots,x_n) \in X_1 \times \dots \times X_n$ we have

$$\|m(\varepsilon)T(x_1,\dots,x_n)\| \le \prod_{i=1}^{n} (\|S_{i,m(\varepsilon)}(x_i)\| + \|x_i\|);$$

hence

$$\|T(x_1,\dots,x_n)\| \le \prod_{i=1}^{n}\left(\frac{1}{\sqrt[n]{m(\varepsilon)}}\, \|S_{i,m(\varepsilon)}(x_i)\| + \frac{1}{\sqrt[n]{m(\varepsilon)}}\, \|x_i\|\right),$$

so

$$\|T(x_1,\dots,x_n)\| \le \prod_{i=1}^{n}\left(\frac{m(\varepsilon)}{\sqrt[n]{m(\varepsilon)}}\, \|S_{i,m(\varepsilon)}\|\, \|S_i(x_i)\| + \varepsilon\|x_i\|\right),$$

and finally

$$\|T(x_1,\dots,x_n)\| \le \prod_{i=1}^{n}(N_i(\varepsilon)\|S_i(x_i)\| + \varepsilon\|x_i\|). \ \blacksquare$$

A careful reading of the last proof shows that we have only used the joint Strong*-to-norm (resp. jointly Right-to-norm) continuity at 0. On the other hand, it is clear that every multilinear operator T satisfying the above condition (1) must be jointly Strong*-to-norm (resp. jointly Right-to-norm) continuous at 0 on bounded sets. We therefore have:

PROPOSITION 7. *Let $X_1,\dots,X_n$ and X be Banach spaces and let*

$$T : X_1 \times \dots \times X_n \to X$$

be a multilinear operator. Then $T|_{B_{X_1}\times\dots\times B_{X_n}}$ is jointly Strong-to-norm (respectively, Right-to-norm) continuous at* 0 *if, and only if, T satisfies the above condition* (1). $\blacksquare$

In this way, given a multilinear operator T, we have four "natural" kinds of continuity:

1) T is jointly $S^*(X_1,X_1^*) \times \ldots \times S^*(X_n,X_n^*)$-to-norm continuous at 0,
2) T is uniformly $S^*(X_1,X_1^*) \times \dots \times S^*(X_n,X_n^*)$-to-norm continuous on bounded sets,

3) T is jointly $S^*(X_1, X_1^*) \times \cdots \times S^*(X_n, X_n^*)$-to-norm continuous on bounded sets,
4) T is jointly $S^*(X_1, X_1^*) \times \cdots \times S^*(X_n, X_n^*)$-to-norm continuous on bounded sets at 0

(and the analogous statements 1′), 2′), 3′) and 4′) for the Right topology). We already know that

$$\begin{tikzcd} 1) \arrow[r] & 2) \arrow[r] \arrow[d] & 3) \arrow[r] \arrow[d] & 4) \arrow[d] \\ 1') \arrow[r, leftrightarrow] & 2') \arrow[r] & 3') \arrow[r] & 4') \end{tikzcd} \tag{2}$$

We have mentioned in Remark 3 that 2) does not imply 1). The following two examples show that 3) does not imply 2), and 4) does not imply 3), respectively. By Remark 3, 3′) does not imply 2′), and 4′) does not imply 3′).

EXAMPLE 8. Let A be a C^*-algebra. We recall that a positive functional $\phi \in A^*$ is said to be *faithful* on A^{**} if $\phi(a) = 0$ implies that $a = 0$ whenever a is a positive element in A^{**}. Let ϕ be a positive faithful functional on A^{**}. Proposition 5.3 in [33] guarantees that the $S^*(A^{**}, A^*)$-topology on $B_{A^{**}}$ is metrised by the norm

$$\|x\|_\phi^2 = 2^{-1}\phi(xx^* + x^*x).$$

In particular, the $S^*(A, A^*)$-topology on B_A is also metrised by the norm $\|\cdot\|_\phi$. On the other hand, Theorem 3.18 in [25] implies that every n-linear form $T : A \times \cdots \times A \to \mathbb{C}$ is quasi-completely continuous, that is, T is jointly sequentially $S^*(A, A^*)$-to-norm continuous, and hence, since the $S^*(A, A^*)$-topology is metrisable on bounded sets, T is jointly $S^*(A, A^*)$-to-norm continuous on the cartesian product of the closed unit balls. Thus, every n-homogeneous scalar polynomial on A is $S^*(A, A^*)$-to-norm continuous on bounded sets.

Let $A = K(\ell_2)$ be the C^*-algebra of all compact operators on ℓ_2. Then A^{**} coincides with $L(\ell_2)$. Let $\phi \in A^*$ denote the functional defined by $\phi(x) = \sum_n \lambda_n (x(h_n) \,|\, h_n)$, where (h_n) is an orthonormal basis of ℓ_2 and $(\lambda_n) \in \ell_1^+$ with $\lambda_n > 0$ for all n. Then ϕ is faithful in A^{**}. According to the above comments, every $T : A \times A \times A \to \mathbb{C}$ is jointly Strong* continuous on bounded sets. However, we have examples of 3-linear forms which are not uniformly Strong* continuous on bounded sets. Indeed, let $P : A \to c_0$ and $Q : A \to \ell_2$ be given by $P(x) = \sum_n x(h_n) h_n \otimes h_n$ and $Q(x) = x(h_1)$. Consider the mapping

$$T : A^3 \to c_0 \times \ell_2 \times \ell_2 \to \mathbb{C},$$
$$(a, b, c) \mapsto (P(a), Q(b), Q(c)) \mapsto (P(a)Q(b) \,|\, Q(c)).$$

The sequence $(x_n) = (h_n \otimes h_n)$ is Strong*-null in A, and $y_n = h_n \otimes h_1$ is in the closed unit ball of A and $T(x_n, y_n, y_n) = 1$, for all n. Corollary 4 implies that T is not uniformly Strong* continuous on bounded sets.

As mentioned before (Remark 3), the Right and Strong* topologies coincide on bounded sets in a C^*-algebra. For this reason the example given above is also valid for the Right topology. We remark that in [11] there is a counterexample for the Right topology on Banach spaces (although the notation is completely different). Finally, it is interesting to contrast the last example with the behaviour in other topologies (see [4]).

The next example is based on an example in [14]. Being again on a C^*-algebra, it is also valid for the Right topology.

Example 9. Let us consider the commutative C^*-algebra c_0. It is known that the Strong** topology of c_0 is metrisable on bounded sets (cf. Remark 8). Let $T : c_0 \times c_0 \to c_0$ be the bilinear operator defined by

$$T(x, y) = x_1 y$$

for every $x = (x_n), y = (y_n) \in c_0$.

Let $(x^m), (y^m)$ be sequences in c_0. Suppose that (x^m) is Strong*-null. Then it is also weakly null, therefore $x_1^m \to 0$. So, for every bounded sequence $(y^m) \subset c_0$ (in particular for every Strong*-null sequence) we have

$$\|T(x^m, y^m)\| \to 0.$$

It follows that T is jointly Strong*-to-norm continuous on bounded sets at 0. However, let $(e^m)_m$ be the canonical basis of c_0, and consider the sequences

$$x^m = e^1 + e^m \quad \text{and} \quad y^m = e^m.$$

Then (x^m) is Strong* convergent to e^1, (y^m) is Strong*-null but

$$\|T(x^m, y^m)\| = \|e^m\| = 1 \nrightarrow 0.$$

Thus T is not jointly Strong*-to-norm continuous on bounded sets.

Remark 10. We conclude this section with an open problem already posed in [25]. Clearly, if the Right and Strong* topologies coincide on bounded sets in a Banach space, then $i) \Leftrightarrow i')$ for all $i = 2, 3, 4$. The converse is also trivially true. That is, if the Right and the Strong* topologies do not coincide on bounded sets in a general Banach space, then $i) \nLeftrightarrow i')$ for all $i = 1, \ldots, 4$. We do not know of any intrinsic characterisation of those Banach spaces for which the Right and Strong* topologies coincide on bounded sets.

3. The setting of C^*-algebras and JB^*-triples. Complex Banach spaces belonging to the classes of C^*-algebras and JB^*-triples satisfy suitable algebraic-geometric axioms which make the above diagram (2) simpler. For example, C^*-algebras and JB^*-triples satisfy Pełczyński's property (V) (cf. [29, Corollary 6], [8] and Remark 10). Furthermore, as a consequence of the *little Grothendieck inequalities* for C^*-algebras and JB^*-triples, prehilbertian seminorms associated to the algebraic structure are enough to bound every operator from a C^*-algebra or a JB^*-triple into a Hilbert space. Indeed, given a positive functional ϕ in the dual of a C^*-algebra A, the law $z \mapsto \|z\|_\phi^2 := \frac{1}{2}\phi(z^*z+zz^*)$ defines a prehilbertian seminorm on A. The little Grothendieck inequality guarantees the existence of a universal constant $G > 0$ such that for each operator T from a C^*-algebra A to a Hilbert space there exists a norm-one positive functional ϕ in A^* such that

$$\|T(z)\| \leq G\|T\| \, \|z\|_\phi$$

for all $z \in A$ (see [32, 15]). Therefore, the (*algebra*) *Strong***-topology* on A is the topology generated by all the seminorms $\|\cdot\|_\phi$, where ϕ is any positive functional in A^*.

Every C^*-algebra belongs to a more general class of complex Banach spaces called JB^*-triples. A JB^**-triple* is a complex Banach space E equipped with a continuous triple product

$$\{\cdot,\cdot,\cdot\} : E \times E \times E \to E, \qquad (x,y,z) \mapsto \{x,y,z\},$$

which is bilinear and symmetric in the outer variables and conjugate linear in the middle one and satisfies:

(a) (*Jordan identity*)

$$L(x,y)\{a,b,c\} = \{L(x,y)a,b,c\} - \{a,L(y,x)b,c\} + \{a,b,L(x,y)c\}$$

for all $x,y,a,b,c \in E$, where $L(x,y) : E \to E$ is the operator given by $L(x,y)z = \{x,y,z\}$;

(b) for each $x \in E$, the map $L(x,x)$ is a hermitian operator with non-negative spectrum;

(c) $\|\{x,x,x\}\| = \|x\|^3$ for all $x \in E$.

Every C^*-algebra is a JB^*-triple with respect to

$$\{x,y,z\} := \frac{1}{2}\,(xy^*z + zy^*x).$$

The Banach space $L(H,K)$ of all bounded linear operators between two complex Hilbert spaces H,K is also an example of a JB^*-triple with respect to $\{R,S,T\} = \frac{1}{2}(RS^*T + TS^*R)$.

When ϕ is any element in the dual of a JB^*-triple, E, and y is a norm-one element in E^{**} such that $\phi(y) = \|\phi\|$, then the mapping

$$x \mapsto \|x\|_\varphi := (\varphi\{x,x,y\})^{1/2} = (\varphi L(x,x)y)^{1/2}$$

induces a prehilbertian seminorm on E whose values are independent of the choice of y. By the little Grothendieck inequality, there exists a universal constant $G > 0$ such that for each operator T from E to a Hilbert space there exist two norm-one positive functionals ϕ_1, ϕ_2 in E^* such that

$$\|T(z)\| \le G\|T\| \, \|z\|_{\phi_1,\phi_2}$$

for all $z \in E$, where $\|z\|_{\phi_1,\phi_2}$ denotes $\sqrt{\|z\|^2_{\phi_1} + \|z\|^2_{\phi_2}}$ (see [5, 20, 23]).

Due to the above reasons, the classes of C^*-algebras and JB^*-triples are appropriate settings to apply Theorem 1 and Corollary 4. There are other reasons to specialise our study to that setting. We shall show that the joint Strong*-to-norm continuity of a multilinear operator can be seen, in this case, as a property of C^*- and JB^*-summability.

Let us recall the concepts of C^*- and JB^*-triple-summing operators. Pisier [32] introduced the following definition: an operator $T : A \to X$ from a C^*-algebra to a Banach space is said to be *q-C^*-summing* if there exists a constant C such that for every finite sequence $(a_1, \ldots, a_n)$ of self-adjoint elements in A,

$$\Big(\sum_{i=1}^{n} \|T(a_i)\|^q\Big)^{1/q} \le C\Big\|\Big(\sum_{i=1}^{n} |a_i|^q\Big)^{1/q}\Big\|, \tag{3}$$

where, for each $x \in A$, we write $|x| = \big(\frac{xx^*+x^*x}{2}\big)^{1/2}$. The smallest constant C satisfying the above inequality is denoted by $C_q(T)$.

The following definition is taken from [21]. Let E be a JB^*-triple and Y a Banach space. An operator $T : E \to Y$ is said to be *2-JB^*-triple-summing* if there exists a positive constant C such that for every finite sequence $(x_1, \ldots, x_n)$ of elements in E we have

$$\sum_{i=1}^{n} \|T(x_i)\|^2 \le C\Big\|\sum_{i=1}^{n} L(x_i, x_i)\Big\|. \tag{4}$$

The smallest constant C for which (4) holds is denoted $C_2(T)$.

We can now define the C^*-algebra and JB^*-triple versions of 2-dominated multilinear operators (see for instance [31]).

Definition 11. Let $A_1, \ldots, A_n$ be C^*-algebras (or JB^*-triples) and let X be a Banach space. A multilinear operator $T : A_1 \times \cdots \times A_n \to X$ is said to be *2-C^*-dominated* (respectively *2-JB^*-triple-dominated*) if there exists a positive constant C satisfying

$$\Big(\sum_{i=1}^{k} \|T(x_i^1, \ldots, x_i^n)\|^{2/n}\Big)^{n/2} \le C\Big\|\Big(\sum_{i=1}^{k} |x_i^1|^2\Big)^{1/2}\Big\| \cdots \Big\|\Big(\sum_{i=1}^{k} |x_i^n|^2\Big)^{1/2}\Big\| \tag{5}$$

for every collection $\{(x_i^j)_{i=1}^k \subset A_j : j = 1, \ldots, n\}$ of finite sequences of self-adjoint elements (respectively, the inequality

$$(6)\qquad \Big(\sum_{i=1}^{k} \|T(x_i^1,\ldots,x_i^n)\|^{2/n}\Big)^n \le C\Big\|\sum_{i=1}^{k} L(x_i^1,x_i^1)\Big\|\cdots\Big\|\sum_{i=1}^{k} L(x_i^n,x_i^n)\Big\|$$

is satisfied for every collection of finite sequences $\{(x_i^j)_{i=1}^k \subset A_j : j = 1,\ldots,n\}$.)

The smallest constant C satisfying the above inequality is denoted by $D_2(T)$.

If T is 2-C^*-dominated, then clearly the elements appearing in (5) can be considered in A_i instead of $(A_i)_{\mathrm{sa}}$ by a simple change in the constant.

Every 2-dominated multilinear operator defined on the cartesian product of n C^*-algebras (respectively, JB^*-triples) is 2-C^*-dominated (respectively, 2-JB^*-triple-dominated), but the converse is in general false (cf. Remark 1.2 in [32]).

Remark 12. Every C^*-algebra A can be equipped with a structure of JB^*-triple with product $\{a,b,c\} := \frac{1}{2}(ab^*c + cb^*a)$. Let $(x_i)_{i=1}^k$ be a finite sequence of elements in A. By [21, Remark 3.2], we have

$$(7)\qquad \Big\|\sum_{i=1}^{n} |x_i|^2\Big\| \le \Big\|\sum_{i=1}^{n} L(x_i,x_i)\Big\| \le 2\Big\|\sum_{i=1}^{n} |x_i|^2\Big\|.$$

Given C^*-algebras $A_1,\ldots,A_n$, a Banach space X, and a multilinear operator $T : A_1 \times \cdots \times A_n \to X$, the inequalities (7) show that T is 2-C^*-dominated if and only if it is 2-JB^*-triple-dominated.

Our next goal is a multilinear extension of Pietsch's factorization theorem for C^*-algebras and JB^*-triples. We shall extend ideas and techniques originated in [32], [10] and [21]. We need some previous results and definitions. A collection Γ of real functions defined on a set K is called *concave* if, given $f_1\ldots,f_m$ in Γ and positive real numbers $\alpha_1,\ldots,\alpha_m$ such that $\sum_{i=1}^m \alpha_i = 1$, there exists $f \in \Gamma$ satisfying $f(x) \ge \sum_{i=1}^m \alpha_i f_i(x)$ for all $x \in K$. It can be easily seen that Γ convex implies Γ concave. The main tool needed later is the following Ky Fan lemma (see, for instance, [30, E.4]):

Lemma 13. *Let K be a compact convex subset of a linear topological Hausdorff space, and let Γ be a concave collection of lower semicontinuous convex real functions f on K. Suppose that for every $f \in \Gamma$ there exists $x \in K$ with $f(x) \le C$ (constant). Then we can find $x_0 \in K$ such that $f(x_0) \le C$ for all $f \in \Gamma$ simultaneously.* ■

Let us also briefly recall some notions pertaining to numerical range. For each norm-one element u in a Banach space X, the *states of X relative to u*, $D(X,u)$, are defined to be the non-empty, convex, and weak*-compact subset of X^* given by

$$D(X,u) := \{\Phi \in B_{X^*} : \Phi(u) = 1\}.$$

For each $x \in X$, the symbol $V(X,u,x)$ will stand for the *numerical range* of x relative to u, that is, $V(X,u,x) := \{\Phi(x) : \Phi \in D(X,u)\}$. The *numerical radius* of x relative to u, $v(X,u,x)$, is given by

$$v(X,u,x) := \max\{|\lambda| : \lambda \in V(X,u,x)\}.$$

It is well known that a bounded linear operator T on a complex Banach space X is hermitian if and only if $V(BL(X), I_X, T) \subseteq \mathbb{R}$ (cf. [7, Corollary 10.13]).

We can now state the desired factorization theorem.

THEOREM 14. *Let $E_1, \ldots, E_n$ be JB^*-triples, let X be a Banach space and let $T : E_1 \times \cdots \times E_n \to X$ be an n-linear operator. The following statements are equivalent*:

(a) *T is jointly $S^*(E_1, E_1^*) \times \cdots \times S^*(E_n, E_n^*)$-to-norm continuous.*
(b) *There exist a positive constant C and norm-one functionals ϕ_i^1, ϕ_i^2 in E_i^* such that*

$$\|T(x^1, \ldots, x^n)\| \le C\|T\|\, \|x^1\|_{\phi_1^1,\phi_1^2} \cdot \ldots \cdot \|x^n\|_{\phi_n^1,\phi_n^2}$$

for every $(x^1, \ldots, x^n) \in E_1 \times \cdots \times E_n$.
(c) *T is 2-JB^*-triple-dominated.*

Proof. (a)$\Rightarrow$(b). By Theorem 1 and Remark 3 statement (a) is equivalent to T factorizing through the cartesian product of n Hilbert spaces. The little Grothendieck inequality for JB^*-triples ensures the existence of a positive constant C and norm-one functionals $\phi_i^1, \phi_i^2 \in E_i^*$ such that

$$\|T(x^1, \ldots, x^n)\| \le C\|T\|\, \|x^1\|_{\phi_1^1,\phi_1^2} \cdot \ldots \cdot \|x^n\|_{\phi_n^1,\phi_n^2}$$

for every $(x^1, \ldots, x^n) \in E_1 \times \ldots \times E_n$, which gives (b).

(b)$\Rightarrow$(c). For $j \in \{1, \ldots, n\}$, let us take a finite sequence $(x_i^j)_{i=1}^k \subset E_j$. Let ϕ_j^1, ϕ_j^2 be the norm-one functionals in E_j^* given by (b) and let z_j^k be a norm-one element in E_j^{**} with $\phi_j^k(z_j^k) = 1$. Then

$$\begin{aligned}
\Big(\sum_{i=1}^k \|T(x_i^1, \ldots, x_i^n)\|^{2/n}\Big)^{n/2} \\
&\le C\|T\| \Big(\sum_{i=1}^k \|x_i^1\|^{2/n}_{\phi_1^1,\phi_1^2} \cdot \ldots \cdot \|x_i^n\|^{2/n}_{\phi_n^1,\phi_n^2}\Big)^{n/2} \\
&\le C\|T\| \Big(\sum_{i=1}^k \|x_i^1\|^2_{\phi_1^1,\phi_1^2}\Big)^{1/2} \cdot \ldots \cdot \Big(\sum_{i=1}^k \|x_i^n\|^2_{\phi_n^1,\phi_n^2}\Big)^{1/2}
\end{aligned}$$

(via Hölder's inequality)

$$= C\|T\|\Big(\phi_1^1\sum_{i=1}^k L(x_i^1,x_i^1)(z_1^1)+\phi_1^2\sum_{i=1}^k L(x_i^1,x_i^1)(z_1^2)\Big)^{1/2}\cdot\ldots$$

$$\ldots\cdot\Big(\phi_n^1\sum_{i=1}^k L(x_i^n,x_i^n)(z_n^1)+\phi_n^2\sum_{1}^k L(x_i^n,x_i^n)(z_n^2)\Big)^{1/2}$$

$$\le \sqrt{2}\,C\|T\|\Big\|\sum_{i=1}^k L(x_i^1,x_i^1)\Big\|^{1/2}\cdot\ldots\cdot\Big\|\sum_{i=1}^k L(x_i^n,x_i^n)\Big\|^{1/2}.$$

(c)⇒(a). For every $1\le j\le n$ we define $K_j := D(L(E_j),\mathrm{Id}_{E_j})$. Clearly, K_j is a weak*-compact subset in $L(E_j)^*$.

Set $K = K_1\times\cdots\times K_n$. For any families $(x_i^1)_{i=1}^k\subset E_1,\ldots,(x_i^n)_{i=1}^k\subset E_n$, we define the convex function $f_{x_i^1,\ldots,x_i^n}: K\to\mathbb{R}$ by

$$f_{x_i^1,\ldots,x_i^n}(\Phi_1,\ldots,\Phi_n) = \sum_{i=1}^k\Big(n\|T(x_i^1,\ldots,x_i^n)\|^{2/n}-D_2(T)^{1/n}\sum_{j=1}^n\Phi_j(L(x_i^j,x_i^j))\Big).$$

Define now the set

$$\Gamma := \{f_{x_i^1,\ldots,x_i^n}: k\in\mathbb{N},\ (x_i^1)_{i=1}^k\subset E_1,\ldots,(x_i^n)_{i=1}^k\subset E_n\}\subset C(K,\mathbb{R}).$$

Let $k_1,k_2\in\mathbb{N}$, $(x_i^1)_{i=1}^{k_1},(y_j^1)_{j=1}^{k_2}\subset E_1,\ldots,(x_i^n)_{i=1}^{k_1},(y_j^n)_{j=1}^{k_2}\subset E_n$, and $0<t<1$. It is not hard to see that $tf_{x_i^1,\ldots,x_i^n}+(1-t)f_{y_j^1,\ldots,x_j^n} = f_{z_l^1,\ldots,x_l^n}\in\Gamma$, where, for each $m=1,\ldots,n$, we define

$$z_1^m,\ldots,z_{k_1+k_2}^m = t^{1/2}x_1^m,\ldots,t^{1/2}x_{k_1}^m,(1-t)^{1/2}y_1^m,\ldots,(1-t)^{1/2}y_{k_2}^m.$$

This shows that Γ is convex and hence concave in the terminology of [30, E.4]. We claim that for every $f_{x_i^1,\ldots,x_i^n}\in\Gamma$ there exists $(\Phi_1^f,\ldots,\Phi_n^f)\in K$ such that $f(\Phi_1^f,\ldots,\Phi_n^f)\le 0$. Indeed, by Sinclair's theorem (see [7, Theorem 11.17]),

$$\|S\| = \sup_{\Phi\in K_j}|\Phi(S)| \tag{8}$$

for every hermitian operator S on E_j. The operator $S_j=\sum_{i=1}^k L(x_i^j,x_i^j)$ is hermitian, thus there exists $\Phi_j^f\in K_j$ such that

$$\Big\|\sum_{i=1}^k L(x_i^j,x_i^j)\Big\| = \Phi_j^f\Big(\sum_{i=1}^k L(x_i^j,x_i^j)\Big) = \sum_{i=1}^k\Phi_j^f(L(x_i^j,x_i^j)).$$

Since $(\Phi_1^f,\ldots,\Phi_n^f)\in K$, we have

$$
\begin{aligned}
&f_{x_i^1,\dots,x_i^n}(\Phi_1^f,\dots,\Phi_n^f)\\
&\qquad = \sum_{i=1}^k \Big(n\|T(x_i^1,\dots,x_i^n)\|^{2/n} - D_2(T)^{1/n} \sum_{j=1}^n \Phi_j^f(L(x_i^j,x_i^j))\Big)\\
&\qquad = \sum_{i=1}^k n\|T(x_i^1,\dots,x_i^n)\|^{2/n} - D_2(T)^{1/n} \sum_{j=1}^n \Big\|\sum_{i=1}^k L(x_i^j,x_i^j)\Big\|.
\end{aligned}
$$

As a consequence of the generalized means inequality (see for instance [16, p. 17]) we know that

$$
n\prod_{j=1}^n b_j^{1/n} \le \sum_{j=1}^n b_j
$$

for every $b_1,\dots,b_n \ge 0$. Therefore

$$
\begin{aligned}
&f_{x_i^1,\dots,x_i^n}(\Phi_1^f,\dots,\Phi_n^f)\\
&\quad \le \sum_{i=1}^k n\|T(x_i^1,\dots,x_i^n)\|^{2/n} - nD_2(T)^{1/n} \prod_{j=1}^n \Big\|\sum_{i=1}^k L(x_i^j,x_i^j)\Big\|^{1/n}\\
&\quad = \sum_{i=1}^k n\|T(x_i^1,\dots,x_i^n)\|^{2/n} - nD_2(T)^{1/n} \Big(\prod_{j=1}^n \Big\|\sum_{i=1}^k L(x_i^j,x_i^j)\Big\|\Big)^{1/n} \le 0.
\end{aligned}
$$

By the Ky Fan lemma there exists an element $(\Phi_1^0,\dots,\Phi_n^0)\in K$ such that $f_{x_i^1,\dots,x_i^n}(\Phi_1^0,\dots,\Phi_n^0)\le 0$ for every $f_{x_i^1,\dots,x_i^n}\in \Gamma$. Thus,

$$
\sum_{i=1}^k n\|T(x_i^1,\dots,x_i^n)\|^{2/n} \le D_2(T)^{1/n} \sum_{i=1}^k \sum_{j=1}^n \Phi_j^0(L(x_i^j,x_i^j))
$$

for any families $(x_i^1)_{i=1}^k \subset E_1,\dots,(x_i^n)_{i=1}^k \subset E_n$. When specialised to the case $k=1$, the above inequality implies that

$$
n\|T(x^1,\dots,x^n)\|^{2/n} \le D_2(T)^{1/n} \sum_{j=1}^n \Phi_j^0(L(x_i^j,x_i^j)) \tag{9}
$$

for every $(x^1,\dots,x^n)\in E_1\times\dots\times E_n$.

We claim that T factors through the cartesian product of n Hilbert spaces. Indeed, for every element x in a JB^*-triple E, the operator $L(x,x)$ is hermitian with non-negative spectrum. In particular, for each state $\Phi\in D(L(E),\mathrm{Id}_E)$, the law $x\mapsto |||x|||_\Phi := (\Phi L(x,x))^{1/2}$ defines a prehilbertian seminorm on E. If we set $N:=\{x\in E : |||x|||_\Phi = 0\}$, then the quotient E/N can be completed to a Hilbert space H_Φ. Let us denote by Q_j the natural quotient map from E_j to $H_{\Phi_j^0}$. Clearly, $\|Q_j(x^j)\| = |||x^j|||_{\Phi_j}$. The claim will follow from the inequality

$$\|T(x^1,\dots,x^n)\| \le D_2(T)^{1/2}\prod_{j=1}^{n} |||x^j|||_{\Phi_j^0}. \tag{10}$$

In order to see the latter we may assume that $T(x^1,\dots,x^n)\neq 0$, otherwise (10) is trivial. If $|||x^{j_0}|||_{\Phi_{j_0}^0}=0$ for some j_0, then $|||\lambda x^{j_0}|||_{\Phi_{j_0}^0}=0$ for every $\lambda>0$. Then (9) gives

$$\lambda^{2/n}n\|T(x^1,\dots,x^n)\|^{2/n}\le\Theta,$$

where Θ is a constant (not depending on λ), which is impossible. Therefore, we may also assume that $|||x^j|||_{\Phi_j^0}>0$ for every $1\le j\le n$. When in (9) we replace x_j with $\overline{x}^j = x^j/|||x^j|||_{\Phi_j^0}$, we get the desired inequality (10). ∎

The appropriate version of the above result in the setting of C^*-algebras now follows from the above theorem together with the little Grothendieck inequality for C^*-algebras.

THEOREM 15. *Let $A_1,\dots,A_n$ be C^*-algebras, let X be a Banach space and let $T: A_1\times\dots\times A_n\to X$ be an n-linear operator. The following statements are equivalent*:

(a) *T is jointly $S^*(A_1,A_1^*)\times\dots\times S^*(A_n,A_n^*)$-to-norm continuous.*
(b) *There exist a positive constant C and norm-one positive functionals ϕ_i in E_i^* such that*
$$\|T(x^1,\dots,x^n)\|\le C\|T\|\,\|x^1\|_{\phi_1}\cdot\ldots\cdot\|x^n\|_{\phi_n}$$
for every $(x^1,\dots,x^n)\in A_1\times\dots\times A_n$.
(c) *T is 2-C^*-dominated.* ∎

Let $T: A_1\times\dots\times A_n\to X$ be a multilinear operator on the cartesian product of n C^*-algebras. Inspired by the definition of multiple summing multilinear operators, we shall say that T is *multiple* 2-C^*-*summing* if there is a positive constant C such that for any $k_1,\dots,k_n\in\mathbb{N}$ and $(x_{i_j}^j)_{i_j=1}^{k_j}\subset A_j$, $1\le j\le n$, we have

$$\Big(\sum_{j=1}^{n}\sum_{i_j=1}^{k_j}\|T(x_{i_1}^1,\dots,x_{i_n}^n)\|^2\Big)^{1/2}\le C\Big\|\Big(\sum_{i_1=1}^{k_1}|x_{i_1}^1|^2\Big)^{1/2}\Big\|\cdots\Big\|\Big(\sum_{i_n=1}^{k_n}|x_{i_n}^n|^2\Big)^{1/2}\Big\|.$$

In the same way, we could define the *absolutely* 2-C^*-*summing* operators.

It is natural to ask whether in Theorem 15, 2-C^*-dominated operators can be replaced with multiple 2-C^*-summing (or absolutely 2-C^*-summing) operators. The following example shows that the answer is, in general, negative.

EXAMPLE 16. By Theorem 3.1 in [6] every trilinear form

$$T:\ell_\infty\times\ell_\infty\times\ell_\infty\to\mathbb{C}$$

is multiple 2-summing and hence multiple 2-C^*-summing. Corollary 4.16 in [9] yields a surjective operator $q : \ell_\infty \to \ell_2$. Let $(b_n)_n \subset \ell_\infty$ be a bounded sequence such that $q(b_n) = h_n$, where (h_n) denotes the canonical basis in ℓ_2. We define $V : \ell_\infty \times \ell_\infty \times \ell_\infty \to \mathbb{C}$, $V(a,b,c) := \sum_{n=1}^{\infty} a_n q(b)_n q(c)_n$, where, for each x in ℓ_∞, $q(x)_n$ denotes the nth coordinate of $q(x)$. We have seen that V is multiple 2-C^*-summing. We claim that V is not 2-C^*-dominated. Indeed, otherwise, by Theorem 15, there would exist a positive constant C and norm-one positive functionals ϕ_1, ϕ_2, ϕ_3 in ℓ_∞^* such that

$$\|V(a,b,c)\| \leq C\|V\| \, \|a\|_{\phi_1} \|b\|_{\phi_2} \|c\|_{\phi_3} \leq C\|a\|_{\phi_1} \|b\| \, \|c\|$$

for all $a,b,c \in \ell_\infty$. Let (e_n) denote the canonical basis of ℓ_∞. It is well known that (e_n) is Strong*-null, thus the above inequality implies

$$1 = \|T(e_n, b_n, b_n)\| \leq \|e_n\|_{\phi_1} \to 0,$$

which is impossible.

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