Entropy of the mixture of sources

Suppose that we are given two sources of information $S_1$, $S_2$ on measurable space $(X, \Sigma)$, represented by the probability measures $\mu_1, \mu_2$. Let $Q$ be an “error-control”. We assume that we lossy-code information from $S_1$ with $Q$-acceptable alphabet $P_1$ and from $S_2$ with $Q$-acceptable alphabet $P_2$.

Consider a new source $S$ which sends a signal produced by source $S_1$ with probability $a_1$ and by source $S_2$ with probability $a_2 = 1 - a_1$. We provide a simple greedy algorithm which constructs a $Q$-acceptable coding alphabet $P$ of $S$ such that the entropy $h(P)$ satisfies:

$$h(S; P) \leq a_1 h(S_1; P_1) + a_2 h(S_2; P_2) + 1.$$ 

In the proof of the above formula the basic role is played by a new equivalent definition of entropy based on measures instead of partitions which we call weighted entropy.

Weighted entropy describes the statistical amount of information needed in the random lossy-coding. Moreover, it provides the computation and interpretation of the entropy with respect to “formal” convex combination $a_1 P_1 + a_2 P_2$, where $P_1, P_2$ are partitions (which clearly does not make sense in the classical approach).

As a consequence we obtain an estimation of the entropy and Rényi entropy dimension of the convex combination of measures. In particular if probability measures $\mu_1, \mu_2$ have entropy dimension then

$$\dim_E(a_1 \mu_1 + a_2 \mu_2) = a_1 \dim_E(\mu_1) + a_2 \dim_E(\mu_2).$$