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On some properties for dual spaces of Musielak-Orlicz function spaces

For every Musielak-Orlicz function Φ we define a convex modular I_Φ by

$$I_\Phi(x) = \int_T \Phi(t, x(t))\, d\mu$$

for every $x \in L^0$. Then the *Musielak-Orlicz* function space L_Φ and its subspace E_Φ are defined as follows:

$$L_\Phi = \{x \in L^0 : I_\Phi(\lambda x) < +\infty \text{ for some } \lambda > 0\},$$
$$E_\Phi = \{x \in L^0 : I_\Phi(\lambda x) < +\infty \text{ for any } \lambda > 0\}.$$

For any $x \in L_\Phi$ the *Luxemburg norm* is defined by

$$\|x\| = \inf\{k > 0 : I_\Phi(x/k) \leq 1\},$$

and the *Orlicz norm* is defined by

$$\|x\|^o = \sup\Big\{\int_T x(t)y(t)\, d\mu : I_\Phi(y) \leq 1\Big\}.$$

Let us note that the Orlicz norm on L_Φ can be also defined by the very useful *Amemiya formula*:

$$\|x\|^o = \inf_{k>0} \frac{1}{k}(1 + I_\Phi(kx)).$$

It is known that any functional $f \in (L_\Phi)^*$, where L_Φ is a Musielak-Orlicz space, is of the form $f = v + \varphi$ ($v \in L_{\Phi^*}$, $\varphi \in F$), where $\varphi \in F$ means that $\langle x, \varphi\rangle = 0$ for any $x \in E_\Phi$ and Φ^* is the Musielak-Orlicz function conjugate to Φ in the sense of Young, as well as that if L_Φ is equipped with the Luxemburg norm, then

$$\|f\|^o = \|v\|^o_{\Phi^*} + \|\varphi\|^o,$$

and if L_Φ is equipped with the Orlicz norm, then

$$\|f\| = \inf\Big\{\lambda > 0 : \rho^*\Big(\frac{f}{\lambda}\Big) \leq 1\Big\},$$

where $\rho^*(f) = I_{\Phi^*}(v) + \|\varphi\|$ for any $f \in (L_\Phi)^*$. We will present relationships between the modular ρ^* and the norm $\|\cdot\|$ in the dual spaces $(L_\Phi)^*$ in the case when L_Φ is equipped with the Orlicz norm.