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Sufficient Optimality Conditions for a Control Problem Linear in a Part of Control Components

The classical sufficient second order optimality conditions for an optimization problem with constraints require that the second variation of Lagrangian be positive definite on the cone of critical directions. In this talk, we present sufficient quadratic optimality conditions of such type for optimal control problems with a vector control variable having two components: a *continuous* unconstrained control *appearing nonlinearly* in the control system and a *bang-bang* control *appearing linearly* and belonging to a convex polyhedron. We give a statement of the control problem with continuous and bang-bang control components, formulations of the maximum principle and the notion of bounded-strong local minimum. We formulate second order sufficient optimality conditions for a bounded-strong minimum in the problem. Using a suitable transformation via a Riccati matrix equation, we develop numerical methods to test the positive definiteness of the quadratic form and give criteria for the positive definiteness of the quadratic form on the critical cone in terms of solutions to a matrix Riccati equation which may be discontinuous at the switching times. The results are applied to an economic control problem in optimal production and maintenance which was introduced by Cho, Abad and Parlar. We show that the numerical solution obtained by Maurer, Kim, and Vossen satisfies our second order test while existing sufficiency results fail to hold. We give another important example of a problem of such type: the planar Earth-Mars transfer with minimal flight time. Moreover, the importance of SSC is due to its crucial role in the sensitivity analysis of parametric optimal control problems. The talk is based on our joint work with Prof. Helmut Maurer (University of Münster, Germany).