Inverse scattering problems with non-over-determined data

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Abstract

From the mid-forties of the last century there were no uniqueness theorems for three-dimensional inverse scattering problems with non-over-determined data. Such theorems are now proved.

First we present the uniqueness theorem for inverse obstacle scattering problem ([7], [8]). Let $A(\beta, \alpha, k)$ be the scattering amplitude, and $A(\beta) := A(\beta, \alpha_0, k_0)$, where α_0 and $k_0 > 0$ are fixed.

Theorem 0 *The surface S of a bounded obstacle and the boundary condition on S are uniquely determined by the data $A(\beta)$ known for all β on an open subset of the unit sphere S^2.*

Let $A(\beta, \alpha, k)$ be the scattering amplitude corresponding to a real-valued compactly supported potential, $\alpha \in S^2$ is the direction of the incident plane wave, $\beta \in S^2$ is the direction of the scattered wave, $k > 0$ is the wave number, S^2 is the unit sphere in R^3.

The Schrödinger equation $[\nabla^2 + k^2 - q(x)]u = 0$, $x \in R^3$, is the governing equation.

For potentials $q \in H_0^\ell$, $\ell > 3$, where H_0^ℓ is the Sobolev space of functions with compact support, the inverse scattering problem with backscattering data has at most one solution. The following uniqueness theorem is proved:

Theorem 1. *If $A_{q_1}(-\beta, \beta, k) = A_{q_2}(-\beta, \beta, k)$ $\forall \beta \in S^2$, $\forall k \in (k_0, k_1)$, and q_1, $q_2 \in H_0^\ell$, $\ell > 3$, then $q_1 = q_2$.*

Here S_1^2 is an arbitrarily small open subset of S^2, and the interval $|k_0 - k_1| > 0$ can be arbitrarily small.

Under the same assumptions on the class of the potentials, the following uniqueness theorem holds:

Theorem 2. *If $A_{q_1}(\beta, \alpha_0, k) = A_{q_2}(\beta, \alpha_0, k)$ $\forall \beta \in S^2$, $\forall k \in (k_0, k_1)$, and for a fixed $\alpha_0 \in S^2$, then $q_1 = q_2$.*

The uniqueness theorems for inverse scattering problems with fixed-energy data are proved in the monograph [6].

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