The Born-Infeld equation: solutions and equilibrium measures

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In this talk, we deal with the following problem

$$\begin{cases} -\operatorname{div}\left(\frac{\nabla\phi}{\sqrt{1-|\nabla\phi|^2}}\right) = \rho, \quad x \in \mathbb{R}^N,\\ \lim_{|x| \to \infty} \phi(x) = 0. \end{cases}$$
(BI)

The equation in (\mathcal{BI}) appears for instance in the Born-Infeld nonlinear electromagnetic theory: in the electrostatic case it corresponds to the Gauss law in the classical Maxwell theory and so ϕ is the electric potential and ρ is an assigned extended charge density.

In the first part of the talk, we discuss existence, uniqueness and regularity of the solution of (\mathcal{BI}) . In the second part, instead, we deal with existence of *equilibrium measures* ρ^* , namely distributions that produce least-energy potentials among all the possible charge distributions, and properties of the corresponding *equilibrium potentials* ϕ_{ρ^*} for (\mathcal{BI}) .

The results have been obtained in joint works with Denis Bonheure (Université libre de Bruxelles, Belgium), Pietro d'Avenia (Politecnico di Bari, Italy) and Wolfgang Reichel (Karlsruher Institut für Technologie, Germany).