A simple variational approach to weakly coupled elliptic systems
Andrzej Szulkin (Stockholm)

Abstract: The elliptic system of 2 equations

$$-\Delta u_i + \kappa_i u_i = \mu_i u_i^3 + 2\lambda u_i u_j^2, \quad i, j = 1, 2, \ i \neq j$$

where \(\Omega\) is a domain in \(\mathbb{R}^N\) has been extensively studied in dimensions \(N \leq 4\). This system appears e.g. in 2-species physical problems (Bose-Einstein condensates with 2 hyperfine states) and in population dynamics. The condition \(\mu_i > 0\) signifies that the interaction of species (or particles) of the same kind is attractive while \(\lambda < 0\) signifies the repulsive interaction of species of different kind. Also various extensions (nonlinearities other than cubic, \(M\) instead of 2 equations) have been recently studied. In this talk we will be concerned with the system of \(M\) equations and nonlinearities which are not necessarily cubic. We introduce a general variational setting and then discuss existence and multiplicity of fully nontrivial (i.e., \(u_i \neq 0\) for all \(i\)) solutions under different hypotheses on \(\Omega\) and \(p\).

This is joint work with Mónica Clapp.