

A simple variational approach to weakly coupled elliptic systems

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Abstract: The elliptic system of 2 equations

$$-\Delta u_i + \kappa_i u_i = \mu_i u_i^3 + 2\lambda u_i u_j^2, \quad i, j = 1, 2, \quad i \neq j$$

where Ω is a domain in \mathbb{R}^N has been extensively studied in dimensions $N \leq 4$. This system appears e.g. in 2-species physical problems (Bose-Einstein condensates with 2 hyperfine states) and in population dynamics. The condition $\mu_i > 0$ signifies that the interaction of species (or particles) of the same kind is attractive while $\lambda < 0$ signifies the repulsive interaction of species of different kind. Also various extensions (nonlinearities other than cubic, M instead of 2 equations) have been recently studied. In this talk we will be concerned with the system of M equations and nonlinearities which are not necessarily cubic. We introduce a general variational setting and then discuss existence and multiplicity of fully nontrivial (i.e., $u_i \neq 0$ for all i) solutions under different hypotheses on Ω and p .

This is joint work with Mónica Clapp.