Abstract

In the thesis we study a recent approach to harmonic functions on metric measure spaces defined via the mean value property. Namely, we investigate three types of functions and related problems: strongly harmonic functions, p-harmonic functions in connections to nonlinear asymptotic mean value property and asymptotically mean value harmonic functions. Our analysis is divided into three settings: weighted Euclidean domains with a norm induced metric, Carnot–Carathéodory groups and doubling metric measure spaces, respectively.

First, we present a characterization of strongly harmonic functions on Euclidean spaces equipped with a weighted Lebesgue measure and a norm induced metric. The necessary condition says, that any strongly harmonic function is a solution to a system of elliptic partial differential equations, where the number of equations in a system depends on the regularity of the weight. The sufficient condition is proved using the Pizzetti formula and shows that every solution to previously described system of equations is strongly harmonic. The result holds for analytic weights. As an outcome of the discussion we obtain the Sobolev/analytic regularity of strongly harmonic functions assuming Sobolev/analytic regularity of the weight, respectively. The discussion is illustrated by distance functions induced by l^p norm for planar domains. We demonstrate the aforementioned system for a smooth weight and $p = 2$ and show, that for a constant weight and $p \in [1, \infty] \setminus \{2\}$ the space of strongly harmonic functions has dimension 8.

In the second part of the dissertation we work with normalized subelliptic p-Laplace equation in Carnot groups. We show a characterization of continuous viscosity solutions via an asymptotic p-mean value property understood in the viscosity sense.

Finally, we investigate asymptotically mean value harmonic functions in locally doubling metric measure spaces. We employ a refined averaging to prove fractional Hajłasz–Sobolev regularity of functions with finite amv-norm and α-Hölder regularity of strongly amv-harmonic functions for all $0 < \alpha < 1$. An outcome of the discussion is local Lipschitz regularity for strongly harmonic functions obtained under weaker set of assumptions than those known in the literature. Moreover, we show that the space of strongly harmonic functions with polynomial growth has finite dimension whenever the measure has δ-annular decay property. Moreover, we prove Blaschke–Privaloff–Zaremba theorem in the Heisenberg group $\mathbb{H}_1$. We also study blow-ups of functions with finite amv-norm proving, that a tangent function at almost every point is strongly harmonic on the tangent space at that point. In the end, we show that amv-harmonic functions on weighted Euclidean domains with locally Lipschitz weights are solutions to an elliptic partial differential equation.