REVIEW ON THE DOCTORAL DISSERTATION "APPLICATIONS OF ALGEBRAIC METHODS IN GEOMETRIC TOMOGRAPHY" BY BARTOLOMIEJ ZAWALSKI

Dear Sir/Madam,

This beautiful dissertation is presented by three main chapters:

I) On star-convex bodies with rotationally invariant sections
II) Differential characterization of quadratic surfaces
III) On separably integrable symmetric convex bodies.

I will briefly review the results chapter by chapter.

Chapter I. The results are directly related to two very interesting problems in classical convexity. The first one is the following.

Problem 1. Let B^n be a Banach space of finite dimension n and let $k \in \mathbb{N}$ be such that $1 < k < n$. If all the k-dimensional subspaces of B^n are isometrically isomorphic to each other, is B^n a Hilbert space?

In 30's of the last century H. Auerbach, S. Mazur and S. Ulam solved the case $n = 3$. At the end of 1960's M. Gromov settled it for n odd. More recently, J. Bracho and L. Montejano obtained several results on a complex version of Problem 1, S. Ivanov, D. Mamaev and A. Nordskova solved the case $n = 4$, and G. Bor, L. Hernández-Lamoneda, V. Jiménez de Santiago and L. Montejano solved the question $n = 4k + 2 \geq 6$, $n \neq 134$. One of the key elements of the proof of the last authors was to show that the hyperplane sections of the unit ball of B^n must be the body of revolution, which prompted the authors to ask

Problem of L. Montejano. Let $K \subset \mathbb{R}^n$, $n \geq 4$, be a convex body containing the origin in its interior. If every hyperplane section of K passing through the origin is a body of affine revolution, is K necessarily a body of affine revolution?

The main result of this chapter gives an affirmative answer to this question, provided the boundary of K is C^3 and K is origin-symmetric. I find this result just brilliant!

The second very interesting open problem, that also was posed about 100 years ago and has to do with Zawalski's result, is

Problem 2. Let $n \geq 3$, let $1 < k < n-1$, and let K and L be two convex bodies in $\mathbb{R}^n$ such that their projections onto every k-dimensional subspace are congruent. Does it follow that K and L are congruent in the ambient space $\mathbb{R}^n$?

There are several partial results related to this problem done by V. Golubyatnikov, S. Myroshnychenko, N. Zhang and others but it is open even in $\mathbb{R}^3$. One of the subproblems of Problem 2 asks a question similar to

Problem. Let $K \subset \mathbb{R}^n$, $n \geq 4$, be a convex body containing the origin in its interior and let $1 < k < n-1$. If every k-dimensional section of K passing through the origin has a fixed $O(k)$-symmetry, what can one say about K?

C. Saroglou showed that if K is origin-symmetric and the sections of K have a symmetry such that the ellipsoid of inertia of the section is a Euclidean ball, then K is a Euclidean ball. The result of Zawalski is another new step towards the solution of the above problem.

Chapter II. In this chapter Zawalski proves two theorems about Sobolev functions f defined on a plane connected open subset. Under the assumption that the Hessian of f is not non-positive, it is shown that f is a weak solution to a certain non-linear system of PDE iff its graph is contained in a quadratic surface.

These results represent their own interest, but, in particular, they appear in connection with problems mentioned above and might be very helpful for the future research.

Chapter III. The results of this chapter are related to another group of very interesting classical questions that goes back to Newton himself. One of these questions asks if ellipsoids are the only *polynomially integrable* convex bodies in $\mathbb{R}^n$ (i.e, such that their volume section function $A_{K,\xi}(t) = \mathrm{vol}_{n-1}(K \cap (\xi^\perp + t\xi))$, where $\xi^\perp + t\xi$ is the hyperplane orthogonal to a unit vector ξ and having distance t from the origin, can be written as a polynomial it t with coefficients depending on ξ), provided n is odd.

A. Koldobsky, A. Merkurjev and V. Yaskin gave an affirmative answer to this question. Another very interesting solution was given by J. Boman.

I must mention that it is not even known if the same is true in all dimensions, provided $A^2_{K,\xi}(t)$ is a polynomial in t with coefficients depending on ξ.

Zawalski considers a class of *locally k-separably integrable* bodies, $1 \leq k < n$, i.e., the bodies for which the volume function $V_{K,H}(t)$ (which is the n-volume of intersection of K with a k-dimensional right hyper-cylinder with base space H, axis $H^\perp$ and radius t) can be locally written as a finite sum of products of the type $a(H)b(t)$.

He proves that if an origin-symmetric convex body with an infinitely smooth boundary is locally k-separably integrable, then $n-k$ is even and K is an ellipsoid or $n-k$ is odd and K is a Euclidean ball.

As I mentioned at the beginning, I find the results of Zawalski incredibly strong and important, and I consider his dissertation outstanding. Definitely it exceeds the requirements of PhD thesis with distinction (summa cum laude).

Sincerely Yours,

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