ON THE THEORY OF s–RIESZ SETS

R. AYOUSH

1. Abstract

In this dissertation we investigate connections of Harmonic Analysis and Geometric Measure Theory. The thesis contains results which concern systematic development of the theory of s-Riesz sets, i.e. a notion introduced in [RW06] in the context of the study of the regularity of vector-valued measures.

Chapter 1 contains an introduction to the topic of the dissertation and describes research methodology. In Chapter 2 we discuss selected classical results concerning connections of Harmonic Analysis and Geometric Measure Theory.

The original results presented in the dissertation come from the following three preprints:

- R. Ayoush, M. Wojciechowski, *On dimension and regularity of vector-valued measures under Fourier analytic constraints*, preprint, submitted.
- R. Ayoush, D. Stolyarov, M. Wojciechowski, *Hausdorff dimension of measures with arithmetically restricted spectrum*, accepted in Annales Academiæ Scientiarum Fennicæ Mathematica
- R. Ayoush, M. Wojciechowski, *Microlocal approach to the Hausdorff dimension of measures*, preprint, submitted.

These preprints are the essential part of Chapters 3-5 (respectively).

In Chapter 3 we focus on the problem proposed in [RW06], concerning the study of the regularity of vector measures subordinated to a bundle $\phi : \mathbb{R}^n \setminus \{0\} \to \mathbb{G}(k, \mathbb{C}^n)$, i.e. measures whose Fourier-Stieltjes transform satisfy $\widehat{\mu}(\xi) \in \phi(\xi)$ for $\xi \neq 0$. The theorems presented there extend the main result of the paper [RW06] (see Theorem 3 therein) and are also related to results from [ADHR19] (Theorem 1.3. and Corollary 1.4. therein).

Chapter 4 contains a new method of estimating the lower Hausdorff dimension of measures based on arithmetical properties of elements of their spectra. It applies to the classical problem of estimating Hausdorff dimension of Riesz products, i.e. measures of the form

$$\mu_{a,q} = \prod_{k=0}^{\infty} \big(1 + a\cos(2\pi q^k x)\big), \tag{1.1}$$

where $q \geq 3$ is an integer and $a \in [-1, 1]$. Our results, for sufficiently big q's and $|a|$ sufficiently close to 1, improve bounds already known from [HR03], [Pey75], [Fan93], [BMP87].

In Chapter 5 we present connections of Hausdorff dimension with Microlocal Analysis. We prove a criterion which gives an estimate of Hausdorff dimension based

2020 *Mathematics Subject Classification.* 28A33, 28B05, 28A78, 31C10, 33C55, 42B10, 43A90.

Key words and phrases. Fourier transform, Hausdorff dimension, rectifiability, Harmonic Analysis of measures, Riesz products, wave front set, pluriharmonic measures.

on the knowledge about the wave front set of a measure. This criterion is applied to Radon measures on the complex sphere and gives results which generalize classical theorems concerning regularity of pluriharmonic measures, due to Aleksandrov and Forelli, proved in [Ale85] and [For74].