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**Referee report on the Habilitation thesis
"Mathematical analysis of selected hydrodynamic models"
by Aneta Wróblewska-Kamińska**

Dr Aneta Wróblewska-Kamińska defended her PhD of Warsaw Univ. in January 2013. Since then, she participated to several national and international research projects. Notably, she obtained a post-doctoral fellowship at Univ. Paris 7 and at the Academy of Sciences of Czech Republic and a Newton International Fellowship at Imperial College in London. The habilitation thesis proposed by Dr Aneta Wróblewska-Kamińska gathers the content of six papers written within this period. Up to one exception, all these papers are concerned with extensions of the so-called "isentropic compressible Navier Stokes equations":

$$\left\{\begin{aligned} \partial_t\rho + \mathrm{div}(\rho u) &= 0 \\ \partial_t(\rho u) + \mathrm{div}(\rho u\otimes u) &= \mathrm{div}(\mathbb{S} - p\mathbb{I}_d) \\ p &= a\rho^\gamma \\ \mathbb{S} &= \mu(\nabla u + \nabla^\top u) + \lambda \mathrm{div} u \mathbb{I}_3 \end{aligned}\right.$$

Here the unknowns (ρ, u, p) represent the density/velocity-field/pressure of a viscous fluid. The symbols μ, λ (resp. γ) are non-negative parameters representing viscosities (resp. pressure exponent). Such models have been thorougly studied in the last 50 years. Several major improvements have in particular been obtained in the last 20 years thanks to the introduction of finite-energy weak solutions which P.L. Lions on the one hand, and E. Feireisl with co-authors on the other hand, proved to exist globally-in-time. This result opened lines of research to refine the notion of solutions as well as study long-time behavior of solutions and several singular limits of the model. This makes the mathematical study of such viscous compressible fluid models a present very active field of research.

In her studies, Dr Aneta Wróblewska-Kamińska envisages the classical issues of existence of solutions to the Cauchy problem as well as singular limits/long-time behavior of solutions when the model is completed with several more realistic effects such as fluid temperature, motion of the fluid container, or non-newtonian behavior of the viscosities. I give now in more details the content of these papers.

First, in a collaboration with Bartłomiej Matejczyk (*Nonlinearity* 2018), she studies a full model including non-Newtonian behavior of the viscosities and thermal effects in a bounded domain $\Omega \subset \mathbb{R}^3$. Precisely, they assume that the stress tensor $\mathbb{S}$ may depend on space-time position as well as fluid temperature θ and fluid-velocity symmetric gradient Du and complete the system with a convection-diffusion equation for the temperature unknown. Such an extension is motivated by experiments on fluids with non-standard rheology in which large variations of θ or fluid strain may imply different viscosity response. One originiality of this study is to consider a more general viscosity law than the classical p-structure where $\mathbb{S}$ is polynomial in Du. The authors tackle the global existence of finite-energy weak solutions when no-slip boundary conditions are prescribed. Beyond the classical compactness method that they apply, the main novelties of the paper are: first, to design a functional framework which enables to handle the nonlinear behavior of $\mathbb{S}$ (which is based here on Musielak-Orlicz spaces), second, to obtain convergence of nonlinear functionals of the gradients of the approximate velocity-fields by the use of monotonicity arguments.

In two papers written in collaborations with Ondřej Kreml, Václav Mácha and Šárka Nečasová (*Journal de Mathématiques Pures et Appliquées* 2018, *Zeitschrift fur Angewandte Mathematik und Physic* 2018), Dr Aneta Wróblewska-Kamińska studies the motion of a Newtonian compressible viscous fluid with temperature in a moving domain. The velocity V of the adiabatic fluid container is given and the authors tackle the global existence of weak solution when full slip boundary conditions are prescribed on the boundaries of the moving container. The two papers propose two ways of handling temperature effects in the system. In both cases, the authors construct weak solutions to the full Navier-Stokes problem with temperature. The key difficulty of the paper is the construction of a suitable approximation scheme on which a compactness method may be performed. An *ad hoc* construction relying on a careful analysis of the tools appearing in the literature is proposed.

In a paper written as single author (*SIAM Journal on Mathematical Analysis* 2017), Dr Aneta Wróblewska-Kamińska tackles the derivation of a Boussinescq-Oberbeck system. Such a model is supposed to describe the motion of an incompressible stratified fluid convecting a diffusive buoyant quantity (such as salt in water). Starting from the full Navier-Stokes-Fourier system with temperature-dependent viscosities in a perforated domain, she obtains the Boussinescq-Oberbeck system in the low-Mach limit when the perforations of the domain scale suitably with the Mach number. This high piece of analysis borrows to the most refined tools of compactness methods complemented with an adaptation of the spectral analyis of acoustic equations in perforated domains.

More recently, Dr Aneta Wróblewska-Kamińska extended her skills to the analysis of fluid-like models coming from the modelling of crowd behavior (*Mathematical Models and Methods in Applied Sciences* 2019). In collaboration with J. Carrillo and E. Zatorska she considers a Navier-Stokes-Poisson like system made of a standard compressible isentropic Navier-Stokes system with a source term in the momentum equation obtained by convolution of the density with an appropriate kernel. A potential as well as a dissipation term is also added. For such a system, the authors prove global existence of weak solutions and analyse the ω-limit sets.

The last paper which is not concerned with compressible systems is written in collaboration with D. Gérard-Varet (*SIAM Journal on mathematical Analysis* 2016). Following the scientific concerns of Dr Aneta Wróblewska-Kamińska, this paper analyses a boundary-layer problem for a non-Newtonian model in which the fluid viscosity is of p-law type. The main motivation of this paper is to consider the solutions to the Stokes equations in a domain with rough boundaries and compute a virtual boundary conditions on a smoothened domain which would encode the influence of the boundaries-roughness. Such a result is obtained by proving that the solution admits an expansion with respect to the size of the roughness which is based on a careful analysis of the boundary layer problem. All this program which is classical in the framework of boundary layer analysis is made more difficult because of the nonlinearities of a non-Newtonian Stokes system.

To summarize, Dr Aneta Wróblewska-Kamińska contributes in a wide area with a unified motivation: to extend analytical results towards more complete and realistic models relevant to applications. Her results follow a modern questioning and participate to the latest knowledge in the analysis of Navier Stokes equations. Her recent interest for crowd modeling furnishes another promising lead for future research. The quality of her research is testified by the high-level of the scientific journals that published her papers. It is also corroborated by a national prize from Gdansk University in 2015 as well as by the support she got in her applications : she was the PI of 2 national and 2 international projects since 2013. Apart from her publications, Dr Aneta Wróblewska-Kamińska presented her results in many national and intenational conferences. I counted around 5 talks per year since 2013 in mean. For all these reasons, I support warmly the habilitation promotion of Dr Aneta Wróblewska-Kamińska .

M. Hillairet

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