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Report on the scientific achievements of Dr. Grigor Sargsyan in the habilitation procedure in discipline mathematics

Dr. Sargsyan obtained his Ph.D. degree in mathematics in the University of California at Berkeley, under supervision of Prof. John Steel, in 2009. For the doctoral dissertation he received prestigious Karp Prize of the Association for Symbolic Logic. Prior to the current habilitation procedure he did not apply for the degree of *doktor habilitowany*.

In years 2009-2012 he worked in the University of California at Los Angeles as an Assistant Adjunct Professor of Mathematics and an NSF Postdoctoral Research Fellow. In years 2012-2021 he worked in Rutgers University as an Assistant Professor of Mathematics (till May 2018) and Associate Professor of Mathematics (from May 2018). Starting from October 2021 he works as a Professor at IMPAN.

As the scientific achievement in habilitation procedure Dr. Sargsyan submitted a habilitation thesis

The Core Model Induction beyond $L(\mathbb{R})$

consisting of the following six papers:

- Covering with Chang models over derived models, Adv. Math. 384 (2021), 21 pp, 200 pts.
- Translation procedures in descriptive inner model theory, in: Foundations of Mathematics, Contemp. Math. 690, Amer. Math. Soc., Providence, RI, 2017, 205-233.

- 3. (joint with Nam Trang) Tame failures of the unique branch hypothesis and models of $AD_{\mathbb{R}} + \Theta$ is regular, J. Math. Logic 16 (2016), 31 pp., 200 pts.
- 4. Covering with universally Baire operators, Adv. Math. 268 (2015), 603-665, 200pts.
- Nontame mouse from a failure of square at a singular strong limit, J. Math. Logic 14 (2014), 47pp, 200 pts.
- (joint with Nam Trang) Non-tame mice from tame failures of the Unique Branch Hypothesis, Canad. J. Math. 66 (2014), 903-923, 100 pts.

The thesis concerns descriptive inner model theory, combining the methods of descriptive set theory and inner model theory to deal with problems motivated by the Gödel's Program.

Gödel's Program follows Gödel's outlook on set theory. Gödel postulated including into the scope of set theory as many sets as possible, so as to eventually settle the major set-theoretic conjectures. This approach considers various large cardinal axioms, with increasing consistency strenth. Thus far it is unsuccessful. On the other hand, other approaches to thinking about various set-theoretical universes are via forcing or determinacy axioms. These different approaches often settle the basic set-theoretical questions (like CH) in opposite ways, undermining the Gödel's Program.

A major motivation for Sargsyan's research is Steel's Program that is a modification of the Gödel's Program. Steel suggests that the various approaches to set-theoretical universe mentioned above may be just various ways of speaking of the same realm of sets (similarly as various languages describe the same human reality). So rather than trying to find the right set theory of the one true Platonistic universe of sets we should rather try showing that various distinct approaches to the universe of sets are in a way bi-translatable. In other words, to show that various natural frameworks for set theory are translatable between each other.

The Sargsyan's thesis deals with two such kinds of frameworks. One is the framework of large cardinal axioms. The other is the framework of determinacy axioms, like AD and its variants. Large cardinal axioms follow the original Gödel's idea of discerning more and more sets. Determinacy axioms contradict the axiom of choice, on the other hand they yield a particularly nice structure of the real line, where all subsets of \mathbb{R} are Lebesgue measurable and have the Baire property. These two frameworks are naturally related, since Martin, Woodin and Steel proved in the 1980's that assuming some large cardinal axioms, $L(\mathbb{R})$ is a model of AD.

The papers from the thesis contain results on constructing inner models of both frameworks of set theory, by the methods of mice and extenders and Core Model Induction (abbreviated as CMI).

Mice and extenders are generalizations of ultraproduct constructions. They are used to obtain inner models satisfying large cardinal axioms, while Core Model Induction is used to obtain models of axioms of determinacy extending $L(\mathbb{R})$.

The main contribution of paper 6 is rendering CMI at some strong cardinal κ , in the context of Unique Branch Hypothesis (namely, assuming that UBH fails for tame trees). In this context, assuming some large cardinal axioms it proves that there is an inner model M of $AD^+ + \theta_0 < \Theta$ (here θ_0 and Θ are cardinals from the Solovay hierarchy).

The same conclusion is reached in Paper 5, under different assumptions (namely, assuming that \Box_{κ} fails at some singular strong limit cardinal κ). The paper does CMI at some $\mu < \kappa$. The main contribution of Paper 5 is adaptation of the method of "branch condensation" from the Ph.D. thesis of Ketchersid to the situation where \Box_{κ} fails.

The main result of Paper 4 is construction (under the assumption of failure of UB-covering) of an inner model of $AD_{\mathbb{R}} + \Theta$ is a regular cardinal. This is the first case where an application of CMI yields such a model.

Paper 3 strengthens the results of Papers 6 and 4, obtaining an inner model of $AD_{\mathbb{R}} + \Theta$ is regular in a new context.

Paper 2 introduces a new translation procedure between mice. This leads to a simplification of the proofs of some earlier results of Woodin and Steel, relating Large Cardinal Hierarchy and Solovay Hierarchy. Also, it answers positively a question of Trevor Wilson, proving that if there is a stationary proper class of Woodin cardinals, then there is a transitive model M of ZFC containing Ord, with a proper class of Woodin cardinals and a strong cardinal.

Paper 1 points out some limitations of CMI and finds some new ways of constructing inner models. Namely, under a large cardinal assumption it proves that for some large cardinal κ there is a transitive model M of $ZFC^$ with height κ^+ such that a particular form of Chang models over derived models (in a generic extension) are models of AD. Here CMI is done below κ . The results of papers 1 - 6 are an important contribution to inner model theory and more generally to mathematics. They improve earlier results and methods of Steel and Woodin. They are published in top logical and mathematical journals.

The applicant has also many other results published in 30 papers (including a monograph), in very good and often top logical and mathematical journals, like Advances Math., Journal of Mathematical Logic, Journal of Symbolic Logic, Transactions AMS. Out of these publications 23 are after obtaining Ph.D. The total IF of these publications is 21.006 (18.617 after Ph.D.) and the total number of ministerial points is 4000 (3360 after Ph.D.). The total number of citations is 53 (without self-citations) and the Hirsch index is 5 (Web of Science).

After getting Ph.D. Dr. Sargsyan carried on his scientific activity in top American universities (the University of California at Los Angeles and Rutgers University) and in IMPAN. He was or is a princial investigator in several grants (3 x NSF, 1 x NCN). He has been invited to numerous conferences, often to give tutorials.

Two of the papers from the habilition thesis have a co-author, Nam Trang. He wrote statements (included in the documentation of the procedure) explaining his contribution to these papers. These statements clearly state that the input of Dr. Sargsyan to these papers was crucial.

Although I do not work myself in the area of Dr. Sargsyan, I was very impressed by his achievements and scientific activity. It is clear to me that he is a mature mathematician pursuing research with sophisticated methods in a difficult part of set theory, attacking and solving hard problems.

In conclusion I may state that the habilitation thesis of Dr. Sargsyan and his other scientific achievements satisfy fully the legal requirements for the scientific degree *doktor habilitowany* in mathematics, in fact they greatly exceed these requirements. I advise the Scientific Council of IMPAN to grant Dr. Sargsyan the degree *doktor habilitowany*.