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Referee report concerning dr Masha Vlasenko's application for habilitation doctorate degree

Dr Masha Vlasenko papers, presented as the habilitation thesis, concern Picard-Fuchs differential equations for families of toric hypersurfaces and modularity of these differential equations. The special interest of her research concerns the arithmetic properties of coefficients of period functions and arithmetic properties of functions that are constructed by the period functions. It is expected that these constructions lead to special elements in p-adic fields which give roots (or poles) of Weil zeta functions of the reduction mod p of the toric hypersurfaces. M. Vlasenko intends to do research in the direction of this conjecture. One of common features of these 5 papers is that the coefficients of period functions are certain coefficients of powers of a Laurent polynomial. Proofs of M. Vlasenko's results are usually technical and apply deep results from geometry, analysis and representation theory. The habilitation thesis of Masha Vlasenko consists of five recent papers:

[1] *Linear Mahler measures and double L-values of modular forms*, J. Number Theory 142 (2014) (joint with Shinder, Evgeny)
[2] *Equations D3 and spectral elliptic curves*, Feynman amplitudes, periods and motives, 135–152, Contemp. Math., 648, AMS (2015) (joint with Golyshev Vasily)
[3] *Dwork's congruences for the constant terms of powers of a Laurent polynomial*, Int. J. Number Theory 12 (2016) (joint with Mellit Anton)
[4] *Higher Hasse-Witt matrices*, Indag. Math. (N.S.) 29 (2018),
[5] *Formal groups and congruences*, Trans. Amer. Math. Soc. 371 (2019)

Below I describe very shortly main results of these papers.

In [1] Vlasenko and Shinder work on logarithmic Mahler measures $m(P)$ of Laurent polynomials $P \in \mathbb{C}[x_1^{\pm 1}, \ldots, x_1^{\pm 1}]$. They are interested in particular in $m_n := m(1 + x_1 + \cdots + x_n)$. For $n = 2$ and $n = 3$ the measure m_n was expressed by simple formulas in terms of special values L-functions and Riemann's zeta function. Such formulas do not exist for $n > 3$. Fernando Rodrigues Villegas stated a conjecture for the expression of m_4 in terms of special values of L-series of special CM modular form. The authors, working on this conjecture, found presentation of m_4 as a linear combination of m_3 and special values of two L-series depending on 3 explicitly given modular forms. This result is not close to the solution of Rodrigues Villegas conjecture but in the spirit of expressing Mahler measures in terms of values of L-functions. The main idea of the proof of this result is the application of iterated integration in computation of m_4 that reduces to computation of m_3.

[2] Based on properties of coefficients of hauptmodules $z(\tau)$ and modular forms $f(\tau)$ as power series in $q(\tau) = e^{2\pi i\tau}$ the main purpose of this paper is to describe modular determinantal differential operators of order 2 and 3. The authors attach to a linear differential operator the corresponding spectral elliptic curve over $\mathbb{Q}$. Based on $z(\tau)$ and $f(\tau)$, they construct special modular form $g(\tau)$ of weight 2 and under assumption that it is new they prove that its associated series L_g is the Hasse-Weil L-function of the above mentioned spectral elliptic curve. Then the coefficients of $g(\tau)$ give a multiplicative arithmetic function which is here the key property for computing the coefficients of modular determinantal differential operators of order 2 and 3.

[3] Sequences of integers (a_n) satisfying Dwork's congruences are rather rare to encounter. The main result in this paper (Theorem 1.1) shows how to generate families of sequences (a_n) of integers satisfying Dwork's congruences. The n-th element of the sequence (a_n) is the constant coefficient of the n-th power $f(X)^n$ of a Laurent polynomial: $f(X) := f(x_1, \ldots, x_m) \in \mathbb{Z}_p[x_1^{\pm 1}, \ldots, x_m^{\pm 1}]$ satisfying some extra assumption on its Newton polytope Δ_f. Further the period function $A(t) := \sum_{n=0}^{\infty} a_n t^n$ is used to construct new function $\alpha(z) := \frac{A(z)}{A(z^p)}$ and its p-adic analytic continuation beyond the unit p-adic open disc to a rigid analytic function on a part of the closed p-adic unit disc. This allows to make an interesting conjecture relating values of α on Teichmüller character and roots (or poles) of Weil zeta function of the reduction mod p of the toric hypersurface $X_t := \{X : t \cdot f(X) = 1\}$.

[4] For Laurent polynomial $f(X) \in R[x_1^{\pm 1}, \ldots, x_m^{\pm 1}]$ over commutative unital ring R and associated Newton polytop Δ_f Masha Vlasenko considers elements $(\beta_n)_{u,v}$ which are coefficients of X^{mu-v} in $f(X)^{n-1}$ with indexes u, v in $\Delta_f^\circ \cap \mathbb{Z}^m$. The main theorem concerns congruences among values of Frobenius lifts σ and derivations δ on matrices β_{p^t}. The congruences lead on the other hand (upon taking limits) to operators F_σ (the Hasse-Witt lift) and N_δ. M. Vlasenko conjectures that F_σ (resp. N_δ) give the Frobenius operator (resp. Gauss-Manin connection) on unit-root crystal attached to f. To make some evidence for her conjecture she explains carefully the action of operators F_σ (resp. N_δ) on modules induced from $H^*_{dR}(X_f)$ and their subquotients and gives a comparison of her results with results of N. Katz.

At the end of her paper M. Vlasenko obtains result on integrality of a formal group law associated to the Laurent polynomial $f(X)$.

[5] In this paper M. Vlasenko continues investigation of integrality of coefficients of formal group laws of dimension 1. A criterion for p-integrality of these coefficients is established via congruences satisfied by the coefficients of power series given by associated invariant differential (Theorem 1 and its Corollary). These results give the opportunity to consider integrality questions concerning two examples of formal groups related to L-functions and to Artin-Mazur and Stienstra formal group laws. On the other hand second general result in this paper (Theorem 2) describes congruences satisfied by the coefficients of the logarithm of an integral formal group law and gives p-adic analytic formula for the characteristic polynomial. This allows to get a criterion for integrality of hypergeometric formal group laws.

Masha Vlasenko shows through her publications that she has a great deal of experience working with the arithmetic and geometric structures related to the Picard-Fuchs differential equations. Three of her five habilitation thesis papers are coauthored with three different coauthors. This shows that she can do research successfully with other mathematicians as well as by herself. The coauthors made clear statements that her contribution was very substantial. Her papers were published in good and very good journals such as: Contemporary Math. AMS, Indag. Math., Int. J. Number Theory, Int. Math. Res. Not., J. of Number Theory, J. Reine Angew. Math., Transactions of AMS. Concerning quotation record, MathSciNet database shows that her papers were quoted 58 times by 67 authors. It is a good record for a habilitation degree application.

Results obtained by Masha Vlasenko, submitted as the habilitation thesis and other results she obtained, are valuable and interesting. Her research achievements satisfy all requirements for awarding her the habilitation doctorate degree. Therefore I support her application for the degree.

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