OPTIMAL CONTROL AND RELATED TOPICS
Czesław Olech in memoriam

Warsaw, 20-21 October 2016

Abstracts

Zvi Artstein (Weizmann Institute)
Olech's extremals in the singular perturbations limit

We shall examine the extent at which Olech's analysis, and theorem, of reaching points at the attainable set of a linear system, can be applied to the limit dynamics of singularly perturbed linear equations.

Alberto Bressan (Penn State University)
Recent trends in differential inclusions

The talk will present some new approaches and recent results on differential equations with multivalued right hand side. In the first part, I shall discuss existence and non-existence results for differential inclusions with upper semicontinuous, non-convex valued right hand side. The main focus of the talk will be on solutions to differential inclusions where the derivative takes values in a set of extreme points. Two different approaches will be considered: a "dual" Baire category method, and a probabilistic approach, replacing residual sets by sets of probability one. A number of open problems will also be discussed.

Paweł Bogdan (Jagiellonian University)
Computational study of polynomial automorphisms (poster)

T. Crespo and Z. Hajto in their joint paper "Picard-Vession theory and the Jacobian problem" (Israel Journal of Mathematics, 186(1):401406, 2012) described a connection between Jacobian Conjecture and Picard-Vessiot theory. They proposed the Wronskian Criterion for checking if a given polynomial mapping is polynomial automorphism. Our first result presented in the poster is a significant simplification of computation necessary to check if a given polynomial map is an automorphism. Second result presented in the poster is an inversion algorithm for polynomial mappings allowing an equivalent statement of the Jacobian Conjecture. In the last part there are presented properties of so called Pascal finite polynomial automorphism.

Pavel Brunovský (Comenius University)
Infinite horizon optimal control, dichotomy and closed range

To establish necessary conditions of optimality for the discrete time infinite horizon optimal control problem one needs that a certain infinite dimensional operator has closed range. It turns out that for the latter dichotomy (a well known concept in dynamical systems) of the dynamics of the problem is a sufficient condition. Also, it will be shown that the assumption is not superfluous.

Arrigo Cellina (University of Milano-Bicocca)
On the existence of solutions to variational problems of slow growth

We discuss the problem of the existence of solutions to variational problems of slow (possibly linear) growth, including the minimal area problem. We present a necessary and sufficient condition for the existence of solutions, for smooth Lagrangians and on smooth domains, without restrictions on the shape of the domain. Joint work with V. Staicu.

Asen Dontchev (Math. Reviews and University of Michigan)
The four theorems of Laurence M. Graves

The classical inverse/implicit function theorems revolve around solving an equation in terms of a parameter and tell us when the solution mapping associated with this equation is a (differentiable) function. Already in 1927 Hildebrandt and Graves observed that one can put aside differentiability obtaining that the solution mapping is just Lipschitz continuous. The idea has evolved in subsequent extensions most known of which are various reincarnations of the Lyusternik-Graves theorem. In the last several decades it has been widely accepted that in order to derive estimates for the solution mapping and put them in use for proving convergence of algorithms, it is sufficient to differentiate what you can and leave the rest as is, hoping that the resulting problem is easier to handle. More sophisticated results may be obtained by employing various forms of metric regularity, of mappings acting in metric spaces aiming at applications to numerical analysis.

Arno van den Essen (University of Nijmegen)
From Olech's Vodka Problem to the theory of Mathieu-Zhao spaces

Many attempts have been made to prove and generalize the Jacobian Conjecture. Almost all generalizations turned out to be false (for example the Markus Yamabe Conjecture, one of Olech's favorites). This led me to the believe that every generalization of the Jacobian Conjecture must be false. However recently, due to work of Wenhua Zhao, a new theory has been created in which several new conjectures have been formulated, which all imply the Jacobian Conjecture. This is the theory of Mathieu-Zhao spaces (MZ-spaces). In this talk I will introduce these spaces, formulate several new conjectures and indicate their relationship with the Jacobian Conjecture.

Hélène Frankowska (CNRS and Univ. Pierre et Marie Curie)
Second Order Necessary Conditions in Optimal Control

This talk is devoted to a second order maximum principle and sensitivity relations for the Mayer problem arising in optimal control theory. Inspired by definitions of tangent and normal cones we introduce second order tangent and normal sets. Next, we apply direct variational methods to derive second order necessary optimality conditions, first in an integral form and, then, deduce from them pointwise conditions. The linearizations of control systems are those learned from C. Olech in 1978.

Andrzej Fryszkowski (Warsaw University of Technology)
Decomposability as a substitute for convexity

A set of functions $K \subset L^1(T, \mathbb{E})$ *is said to be decomposable if for all* $u, v \in K$ *and measurable* $A \subset T$ *the function* equal to u on A and v on the complement A',

$$u \cdot \chi_A + v \cdot (1 - \chi_A) = \begin{cases} u(t) & for \quad t \in A \\ v(t) & for \quad t \in A' \end{cases},$$

belongs to K. The decomposable sets were introduced by Rockafellar and the theory of them was developing by many researchers, including professor Olech.

In this talk I am going to describe the most important properties of such sets, concentrating on their similarity to convex sets and pointing out some Olech's results. I will also discuss their importance in examination of differential inclusions with nonconvex right-hand sides.

Bronisław Jakubczyk (Institute of Mathematics PAS)
Olech's theorems on global stability and models of economic growth

We will present early results of Olech obtained during his stay in the RIAS, a center created by Solomon Lefschetz in Baltimore. They include sufficient conditions for global stability of dynamical systems in the plane, related to so called Markus-Yamabe conjecture, and the Olech-Hartman theorems in higher dimensions. Later applications of these theorems in the analysis of models of economic growth will be discussed.

Anna Ochal (Jagiellonian University)
Optimal Control for Doubly Nonlinear Evolutionary Inclusions (poster)

We study the optimal control of systems for a class of nonlinear hemivariational inequalities which are in the form of evolutionary inclusions involving Clarke's generalized gradient. The control variables are introduced both in the generalized gradient and in source terms. We present the existence of weak solutions to nonlinear inclusions, we prove the upper semicontinuity property of their solution sets and we show the existence of optimal admissible state-control pairs. Finally, we present some examples of our abstract results which appear in applications.

References: L. Gasinski, S. Migorski, A. Ochal, Z. Peng, Optimal Control for Doubly Nonlinear Evolutionary Inclusions, submitted to JOTA, 2016.

Luong V. Nguyen (Institute of Mathematics PAS)
Sensitivity Relations and Regularity of the Minimum Time Function (poster)

Let $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ be a Lipschitz continuous sublinear multifunction and $\mathcal{K}$ be a closed subset of $\mathbb{R}^n$. The minimum time function associated to the target $\mathcal{K}$ for the differential inclusion

$$\begin{cases} \dot{x}(t) & \in & F(x(t)), \qquad \text{a.e. } t > 0 \\ x(0) & = & x_0 \in \mathbb{R}^n \end{cases} \tag{1}$$

is defined as: for $x_0 \in \mathbb{R}^n$,

$$T(x_0) := \inf\{t > 0 : \exists\, x(\cdot) \,\text{satisfying}\,(1)\,\text{with}\, x(0) = x_0 \text{ and } x(t) \in \mathcal{K}\}.$$

In the poster, we present some recent results on sensitivity relations and regularity of the minimum time function. We also give an invariant result for the set of non-Lipschitz points of the function T.

R. Tyrrel Rockafellar (University of Washington)
Progressive Decoupling of Linkages in Variational Inequality Problems with Elicitable Monotonicity

Variational inequalities provide a versatile model for expressing conditions of optimality and equilibrium in many situations, but solution methods typically rely on the global presence of maximal monotonicity. Another difficulty comes from linkage constraints that force interactions between subproblems which otherwise might be solved in parallel. This talk will describe a new procedure, the progressive decoupling algorithm, which iteratively relaxes linkages and is able to determine a solution through local convergence under just a localized property of maximal monotonicity. Such localized monotonicity can moreover often be elicited from the problem structure by an augmentation device.

Tadeusz Rzeżuchowski (Warsaw University of Technology)
Semicontinuity and continuity in differential inclusions – Olech's theorem with mixed assumptions

I shall recall briefly the early history of the theory of existence of solutions of differential inclusions $\dot{x} \in F(t,x)$ (where $x \in \mathbb{R}^n$), in which the situation and necessary assumptions when $F(t,x)$ need not be convex differ much from the one with $F(t,x)$ convex.

The Olech's theorem mentioned in the title puts together both cases, namely the mappings $x \to F(t,x)$ are assumed to be upper semicontinuous and moreover continuous at those x for which $F(t,x)$ is not convex (plus measurability of $t \to F(t,x)$ and boundedness $|F(t,x)| \leqslant \eta(t)$ with η integrable). The investigation of this problem was continued later by several authors.

The original proof given by Olech contained several gaps related to some subtle problems of measurability. It seems that the correction, which was being prepared by Olech himself, has never been published. I shall try to describe what was the nature of these difficulties and, using Olech's notes, the way to overcome them.

Richard Vinter (Imperial College)
Optimal Control Problems with Time Delays

This talk concerns necessary conditions for optimal control problems involving time delays. Time delays are encountered in many areas of control engineering, including process control where they are associated with transport delays of reagents flowing between reactors, and control applications in planetary exploration where communications delays are important factors. Necessary conditions for time delay problems, in the form of a generalized Pontryagin Maximum Principle, go back to the early days of optimal control theory. But a number of important issues have until recently remained unresolved; these concern, in particular, optimality conditions covering free end-time problems, optimality conditions for problems involving non-commensurate delays in the control, and versions of the necessary conditions valid for non-smooth data.

We provide an overview of the area, including an account of recent advances. We also compare and contrast the two main approaches to deriving necessary conditions, one based on the derivation of Lagrange multiplier rules for abstract optimization problems associated with Gamkrelidze, Milyutin and their collaborators, and the other based on perturbation and the application of variational principles, associated with Clarke and Rockafellar.

Jerzy Zabczyk (Institute of Mathematics, PAS)
Controllability with vanishing energy

The talk is concerned with linear control systems which, in the simplest case, are of the form

$$y'(t) = Ay(t) + Bu(t), \quad y(0) = x \in R^n, \ t \geqslant 0, \tag{1}$$

where A and B are $n \times n$ and $n \times m$ matrices and the control function u takes values in R^m. The control system is controllable with vanishing energy if for arbitrary $x, y \in R^n$ and arbitrary $\epsilon > 0$, there exists a control function u and time $T > 0$ such that for the corresponding solution y of (1):

$$y(T) = y, \quad \text{and} \int_0^{\mathrm{T}} |\mathrm{u}(\mathrm{s}|^2 \mathrm{ds} < \epsilon.$$

System (1) is said to be null controllable with vanishing energy if the above property holds for arbitrary x and $y = 0$.

We will present several characterizations of controllable systems with vanishing energy both for finite and infinite dimensional systems. A specific example will be treated with some detail. An interplay between the controllability with vanishing energy and the Liouville theorem on harmonic functions will be discussed as well.

The results were obtained in collaboration with L. Pandolfi and E. Priola.

Henryk Żołądek (University of Warsaw)
An application of the Newton-Puiseux charts in the Jacobian problem

We study 2-dimensional Jacobian maps using so-called NewtonPuiseux charts. These are multi-valued coordinates near divisors of resolutions of indeterminacies at infinity of the Jacobian map in the source space as well as in the target space. The map expressed in these charts takes a very simple form, which allows us to detect a series of new analytical and topological properties.