Some results on recurrence structure of real and complex bivectors in space-times

Bahar Kırık

Istanbul, Turkey

baharkirik@gmail.com

Abstract

The purpose of this study is to investigate the recurrence structure of bivectors (second order skew-symmetric tensors) on $4-$dimensional manifolds having a metric of Lorentz signature. First of all, this structure is examined for real bivectors by analysing parallel (or scalable to parallel) or proper recurrence for such tensor fields. After that, this examination is extended to complex bivectors on these manifolds. Some remarks and theorems are given regarding the existence of a real, null, recurrent vector field. All these results are associated with the possible holonomy algebras for Lorentz manifolds. Conditions are expressed regarding the possibility that the manifold admits a properly recurrent bivector and a parallel bivector. A brief review is also done for second order symmetric, recurrent tensor fields for $4-$dimensional manifolds admitting a metric of Lorentz, neutral or positive definite signature.

Some References

[1] W. Ambrose, I. Singer, *A theorem on holonomy*, Trans. Am. Math. Soc. **75** (1953), 428.

[2] G. S. Hall, *Symmetries and Curvature Structure in General Relativity*, World Scientific, 2004.

[3] G. S. Hall, *Recurrence conditions in space-time*, J. Phys A. **10(1)** (1977), 29–42.

[4] G. S. Hall, *Some general, algebraic remarks on tensor classification, the group $O(2,2)$ and sectional curvature in $4-$dimensional manifolds of neutral signature*, to appear in Colloq. Math.

[5] G. S. Hall, *Covariantly constant tensors and holonomy structure in general relativity*, J. Math. Phys. **32(1)** (1991), 181–187.

[6] G. S. Hall, *Some comparisons of the Weyl conformal tensor and its classification in 4-dimensional Lorentz, neutral or positive definite signatures*, to appear in Int. J. Mod. Phys. D. DOI:10.1142/S0218271817410048174100481741004.

[7] G. S. Hall, B. Kırık, *Recurrence structures in 4-dimensional manifolds with metric of signature $(+,+,-,-)$*, J. Geom. Phys. **98** (2015), 262–274.

[8] G. S. Hall, Z. Wang, *Projective structure in 4-dimensional manifolds with positive definite metrics*, J. Geom. Phys. **62** (2012), 449–463.

[9] S. Kobayashi, K. Nomizu, *Foundations of Differential Geometry*, Interscience, (vol 1), New York, 1963.

[10] R. K. Sachs, *Gravitational waves in general relativity. VI. the outgoing radiation condition*, Proc. Roy. Soc. London, Series A **264** (1961), 309–338.

[11] J. F. Schell, *Classification of four-dimensional Riemannian spaces*, J. Math. Phys. **2** (1961), 202–206.

[12] A. G. Walker, *On Ruse's spaces of recurrent curvature*, Proc. London Math. Soc. **52** (1950), 36–64.

[13] Z. Wang, G. S. Hall, *Projective structure in $4-$dimensional manifolds with metric of signature $(+,+,-,-)$*, J. Geom. Phys. **66** (2013), 37–49.