Ridigity of Ricci Solitons with Weakly Harmonic Weyl Tensors

Gabjin Yun
Myong Ji University

Abstract

A complete Riemannian metric g on a smooth manifold M^n is called a *gradient Ricci soliton* if there exist a constant ρ and a smooth function f on M satisfying

$$\mathrm{Ric}_g + \mathrm{Hess} f = \rho g, \tag{1}$$

where Ric_g is the Ricci tensor of the metric g, and $\mathrm{Hess} f$ denotes the Hessian of f. A gradient Ricci soliton satisfying (1) is said to be *shrinking, steady* or *expanding* according as $\rho > 0, \rho = 0$, or $\rho < 0$, respectively.

In this talk, we will show some rigidity results on gradient shrinking(or steady) Ricci solitons with weakly harmonic Weyl curvature tensor. We say that a gradient Ricci soliton (M, g) satisfying (1) has weakly harmonic Weyl curvature tensor if $\delta\mathcal{W}(\cdot, \cdot, \nabla f) = 0$, where $\mathcal{W}$ denotes the Weyl curvature tensor.

First, we will show that if a compact gradient shrinking Ricci soliton (M, g) has weakly harmonic Weyl curvature tensor, then (M, g) is Einstein. In the case of noncompact, we will prove that if M is complete gradient shrinking Ricci soliton with weakly harmonic Weyl curvature tensor, then M is rigid in the sense that M is given by a quotient of product of an Einstein manifold with Euclidean space. These are generalizations of the previous known results. Finally, we will show that if (M^n, g) be a complete noncompact gradient *steady* Ricci soliton with weakly harmonic Weyl curvature tensor, and if the scalar curvature s_g attains its maximum at some interior point, then either (M, g) is flat or isometric to the Bryant soliton.