Curvature and 4-dimensional Manifolds.

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In this talk I will discuss the curvature structure of a 4-dimensional manifold with metric g, associated Levi-Civita connection ∇, holonomy group Φ, curvature tensor $Riem$, sectional curvature function σ and Weyl conformal tensor C. I will concentrate on the case when g has neutral signature since the other signatures are known. A brief review will be given of the isometry group $O(2,2)$ associated with g and the holonomy subalgebras of $o(2,2)$. Some strong relationships between g, ∇, Φ and σ and also between C and the conformal class of g will be briefly described, for example, that, except in some very special cases, σ uniquely determines g and C uniquely determines the conformal class of g. This will involve the introduction of the *curvature/Weyl map*. The rest of the talk will involve a classification of $Riem$ using the curvature map and will concentrate on the extent to which $Riem$ determines g and ∇. A simple corollary concerning the symmetries of $Riem$ then follows.