On conformal Killing and harmonic forms on Riemannian symmetric spaces

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Conformal Killing forms have been defined on Riemannian manifolds more than forty-five years ago by Tachibana (see [1]) as a natural generalization of conformal Killing vector fields. Surveys of the publications on these forms can be found in the introduction to our last paper [2].

A Riemannian globally symmetric space of *non-compact type* (M, g) is complete and also (M, g) has a nonpositive sectional curvature. We also know that a Riemannian symmetric space has nonpositive (resp. non-negative) curvature operator if and only if it has nonpositive (resp. non-negative) sectional curvature (see [3]). Note that symmetric spaces of non-compact type are non-compact. After the above remarks, the assertion of the following theorem becomes obvious.

Theorem 1. *A globally symmetric space of non-compact type (M, g) with infinite volume $\mathrm{Vol}_g(M)$ does not admit a nonzero conformal Killing L^2 -form.*

It is well known that a Riemannian globally symmetric space of *compact type* (M, g) is compact and also (M, g) has a nonpositive sectional curvature. Then the following theorem holds.

Theorem 2. *A globally symmetric space of compact type (M, g) does not admit a non-parallel harmonic form.*

References

[1] Tachibana S., On conformal Killing tensor in a Riemannian space, Tohoku Math. Journal **21** (1969) 56-64.

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[5] Duchesne B., Infinite dimensional Riemannian symmetric spaces with fixed-sing curvature operator, Ann. Inst. Fourier, Grenoble **65** (2015), no. 1, 211-244.