Quasi-Einstein metrics and affine structures

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ABSTRACT

A pseudo-Riemannian manifold (M, g) *of dimension* n *is called quasi-Einstein if there is a solution* $f \in \mathcal{C}^\infty(M)$ *of the equation*

$$\mathrm{Hes}_f + \rho - \mu\, df \otimes df = \lambda g \tag{1}$$

for some μ *and* $\lambda \in \mathbb{R}$*, where* Hes_f *and* ρ *denote the Hessian of* f *and the Ricci tensor, respectively [3, 4]. This class of manifolds includes, as particular cases, Einstein metrics and gradient Ricci solitons (see [1]). For* $\lambda = 0$*, Equation* (1) *has a corresponding equation in affine geometry. Let* (Σ, D) *be an affine manifold with torsion-free connection* D*, and set*

$$\mathrm{Hes}^D_{\hat f} + 2\,\rho^D_{\mathrm{sym}} - \mu\, d\hat f \otimes d\hat f = 0, \tag{2}$$

where $\mathrm{Hes}^D_{\hat f} = Dd\hat f$ *is the affine Hessian of* $\hat f$ *and* ρ^D_{sym} *is the symmetric part of the Ricci tensor of* (Σ, D).

The two equations above are linked as follows [2]. If $\hat f$ *is a solution to Equation* (2)*, then the cotangent bundle* $T^*\Sigma$ *of* Σ *equipped with a deformed Riemannian extension* $g_{D,\Phi}$ *is quasi-Einstein for* $f = \pi^* \hat f$ *and* $\lambda = 0$*, where* $\pi : T^*\Sigma \to \Sigma$ *is the projection. Conversely, a self-dual quasi-Einstein manifold which is not locally conformally flat is locally isometric to* $(T^*\Sigma, g_{D,\Phi})$ *and the potential function* f *is the pull-back to* $T^*\Sigma$ *of a solution of* (2) *in* (Σ, D).

The purpose of the talk is to detail the above correspondence and to show some affine properties related to the solutions of Equation (2) *above.*

References

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