HARNACK INEQUALITIES FOR EVOLVING HYPERSURFACES

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(joint work with Paul Bryan and Mohammad N. Ivaki)

Let N^{n+1} be a Riemannian or Lorentzian manifold. Let a smooth, time-dependent family of embeddings $x\colon (0,T)\times M^n \to N^{n+1}$ of a closed, orientable and spacelike manifold M^n move according to the *curvature flow*

$$\dot{x} = -\sigma F \nu, \tag{1}$$

where F depends monotonically on the Weingarten operator of the hypersurface $M_t = x(t,M)$ and ν is a normal vector field along M_t with signature σ.

If the flow hypersurfaces are strictly convex, we introduce a new method to obtain so-called *differential Harnack inequalities* of Li-Yau-type, compare [4], of the form

$$\partial_t F - b(\nabla F, \nabla F) + \frac{p}{(p+1)t}F \geq 0, \tag{2}$$

where $p \neq 0$ is the homogeneity of F and b is the inverse of the second fundamental form. Until today, such estimates are known mostly for flows in the Euclidean space, for example for the mean curvature flow [3] and more general speeds [1]. In the sphere we obtained such estimates for a broad class of speeds [2]. If the flow is viewed in the form (1), the necessary computations are quite heavy. In [1] Andrews has used a reparametrisation by the Gauss map of a strictly convex hypersurface, which simplified the calculations tremendously. However such a map is generally not available in other manifolds.

In this talk we present a substitute for the reparametrisation of (1) by Gauss map, which simplifies the involved calculations in any Riemannian or Lorentzian manifold. We recover (and extend) the known results in the Euclidean case and in the sphere and obtain new Harnack inequalities for the mean curvature flow in locally symmetric Einstein manifolds and for more general flows in the De Sitter space. Via a duality method we obtain so-called *pseudo Harnack inequalities* for flows in the hyperbolic space, which until today tenaciously refused to allow the derivation of such results.

References

[1] Ben Andrews, *Harnack inequalities for evolving hypersurfaces*, Math. Z. **217** (1994), no. 1, 179–197.

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[4] Peter Li and Shing-Tung Yau, *On the parabolic kernel of the Schrödinger operator*, Acta Math. **156** (1986), no. 1, 153–201.