Lagrangian submanifolds in the homogeneous nearly Kähler $S^3 \times S^3$ related to classical differential equations.

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Nearly Kähler manifolds have been studies intensively in the 1970's by Gray. These nearly Kähler manifolds are almost Hermitian manifolds for which the tensor field ∇J is skew-symmetric. In particular, the almost complex structure is non-integrable if the manifold is non-Kähler. A well known example is the nearly Kähler 6-dimensional sphere, whose complex structure J can be defined in terms of the vector cross product on $\mathbb{R}^7$. Recently it has been shown by Butruille that the only homogeneous 6-dimensional nearly Kähler manifolds are the nearly Kähler 6-sphere, the nearly Kähler $S^3 \times S^3$, the projective space $\mathbb{C}P^3$ and the flag manifold $SU(3)/U(1) \times U(1)$. All these spaces are compact 3-symmetric spaces

There are two natural types of submanifolds of nearly Kähler (or more generally, almost Hermitian) manifolds, namely almost complex and totally real submanifolds. Totally real submanifolds are those for which the almost complex structure maps tangent vectors to normal vectors. A special case occurs when the dimension of the submanifold is half of the dimension of the ambiant space. In that case such submanifolds are called Lagrangian and J interchanges the tangent and the normal space.

In this talk we study Lagrangian submanifolds of $S^3 \times S^3$. The first examples of such Lagrangian submanifolds were due to Schäfer and Smoczyk. Other examples have been recently discovered by Moroianu and Semmelmann. In this talk we study Lagrangian submanifolds for which the projection on the first component has nowhere maximal rank. We show that then the projection is necessarely a minimal surface. Conversely we show that starting from a minimal surface and any solution of another classical PDE we can construct a Lagrangian immersion.