Variational problems for a manifold equipped with a distribution

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A distribution $\mathcal{D}$ on a manifold M appears in various situations, e.g. tangent bundle of a foliation or kernel of a differential form. We discuss two curvature related functionals on the space of metrics on $(M, \mathcal{D})$.

1. The mixed scalar curvature $\mathrm{S}_{\rm mix}$ is the simplest invariant of a metric on $(M, \mathcal{D})$. For a stably causal (e.g. globally hyperbolic) spacetime, which is naturally endowed with a codimension-one distribution, the total $\mathrm{S}_{\rm mix}$ is an analog of Einstein-Hilbert action. We show that the Euler-Lagrange equations for any $(M, \mathcal{D})$ look like Einstein field equations with the new Ricci type curvature.

2. Given M^3 equipped with a plane field $\mathcal{D}$ and a vector field T transverse to $\mathcal{D}$, we use 1-form ω such that $\mathcal{D} = \ker\,\omega$ and $\omega(T) = 1$ to construct a 3-form analogous to the Godbillon-Vey class of a foliation. For a metric g on M, we express this form in terms of geometry of $\mathcal{D}$ and the curvature and torsion of its normal curves and derive Euler-Lagrange equations of associated action.

References

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