Compact Riemannian four-manifolds with harmonic curvature

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We describe a step towards a classification of compact four-dimensional Riemannian manifolds whose curvature tensor R is harmonic as a 2-form valued in 2-forms or - equivalently - whose Ricci tensor satisfies the Codazzi equation. The known examples of such manifolds form five (non-disjoint) classes, in which the metric is, respectively, Einstein; conformally flat with constant scalar curvature; locally reducible (of types $1+3$ or $2+2$); and a $2+2$ warped product. This talk presents work in progress, joint with Paolo Piccione, showing how the question of classifying compact four-manifolds with harmonic curvature that lie outside of the five classes named above is reduced to a problem in real algebraic geometry.