Diameter Bounds, Gap Theorems, and Hitchin-Thorpe Inequalities for Compact Quasi-Einstein Manifolds

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In this poster, stimulated by Fernández-López and García-Río, we will give some lower diameter bounds for non-trivial compact shrinking quasi-Einstein manifolds in terms of the Ricci and scalar curvatures. As an application, we will give a gap theorem for compact shrinking quasi-Einstein manifolds by showing a necessary and sufficient condition for the manifolds to be Einstein. Moreover, following recent work by Ma and by Brasil, Costa, and Ribeiro Jr., we will provide a sufficient condition for four-dimensional compact shrinking quasi-Einstein manifolds to satisfy the Hitchin-Thorpe inequality, and to be isometric to the standard four-dimensional sphere $\mathbb{S}^4$. Finally, using a classification of compact shrinking quasi-Einstein manifolds by Catino, we will prove that under a simple condition, a four-dimensional compact shrinking quasi-Einstein manifold with harmonic self-dual part of the Weyl tensor must be isometric to the four-dimensional sphere $\mathbb{S}^4$ or the complex projective surface $\mathbb{CP}^2$ with their canonical metrics, or must be Kähler-Einstein, or its universal covering must be globally conformally equivalent to the standard four-dimensional sphere $\mathbb{S}^4$.