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Topology of dynamics of a nonhomogeneous rotationally symmetric ellipsoid on a smooth plane.

Let us consider an ellipsoid of revolution moving on a smooth horizontal plane under the action of gravity. We construct topological invariants for this system and classify corresponding Liouville foliations up to Liouville equivalence. Two systems are called equivalent if they have the same closure of integral trajectories of systems solutions. Suppose that the mass distribution in the ellipsoid is such that it has an axis of dynamical symmetry coinciding with the axis of geometric symmetry. Moments of inertia about principal axes of inertia perpendicular to symmetry axis are equal to each other. We also assume that the center of mass lies on this symmetry axis (as in the Lagrange top) at distance s from the geometric center of the body.

A free rigid body has six degrees of freedom. We need three coordinates to describe the position of an arbitrary point in the body (e.g., the center of mass) with respect to a fixed space frame, and three more coordinates to describe the orientation of principal axes.

In our case, there is one holonomic constraint: the height of the center of mass above the plane is determined by the orientation of principal axes. Thus, the number of degrees of freedom is reduced to five. Let us write the equation in Eulers form using $f' = \{f; H\}$, where H is the Hamiltonian, and $\{,\}$ is the Poisson bracket on $e(3)^*$. Then in standard (S, R) coordinates we get the following first integrals: $H = \frac{1}{2}\Sigma\frac{S_i^2}{A_i} + U$, where U is the potential energy and A is a constant, and $K = S_3$.

Using the Fomenko-Zieshang invariants, we prove the following theorem.

Theorem. The Liouville foliation associated with the above-described problem can be embedded in the foliation corresponding to the Zhukovsky system describing a heavy gyrostat.

Note that N. E. Zhukovsky (1899) found a generalization of Eulers integrable case, with Hamiltonian $H = \frac{1}{2}\Sigma\frac{(S_i+\lambda_i)^2}{A_i}$. The additional integral is the same as in the Eulers case: $K = S_1^2 + S_2^2 + S_3^2$.