

An algebraic characterization of smooth vector fields using some universal property of $(\Omega^1(M), d)$

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Abstract

We present a new approach to the well known algebraic characterization of smooth vector fields. We show that isomorphism $\text{Der}_{\mathbb{R}}(C^\infty(M)) \cong \mathfrak{X}(M)$ of $C^\infty(M)$ -modules can be recovered using some universal property of $(\Omega^1(M), d)$. More precisely, we use the fact that $(\Omega^1(M), d)$ is the universal derivation in the category of geometric $C^\infty(M)$ -modules.

We also shed new light on this universality of $(\Omega^1(M), d)$ by showing that it is a consequence of Hadamard's Lemma. Some simple ideas behind considering geometric $C^\infty(M)$ -modules we cover as well.