Some potential analysis on homothetic solutions to Mean Curvature Flow and Inverse Mean Curvature Flow

Vicente Palmer
Universitat Jaume I of Castellón
(Work in collaboration with V. Gimeno)

Abstract

We study the two simplest and classical elliptic PDE equations, namely, Laplace's and Poisson's equation defined on Self-shrinkers and Self-expanders for the MCF and Solitons, (with positive or negative velocity constant), for the IMCF. A classical question in this field consists in to explore the existence of bounded and non-constant subharmonic functions defined in the (sub)-manifold , $X_0 : \Sigma^n \to \mathbb{R}^{n+p}$, (i.e., its parabolicity). We have proved that Self-expanders $X_0 : \Sigma^n \to \mathbb{R}^{n+p}$ for the MCF with $n \geq 3$ and Solitons $X_0 : \Sigma^n \to \mathbb{R}^{n+p}$ for the IMCF with negative velocity constant and $n \geq 2$ or positive velocity constant $c > \frac{1}{n-2}$ and $n \geq 3$ are non-parabolic.

If we consider, in some sense, the minimal submanifolds in Euclidean space as a "limit" case of Self-expanders, the non-parabolicity of Self-expanders is then coherent with non-parabolicity of minimal submanifolds $X : \Sigma^n \to \mathbb{R}^{n+p}$ with $n \geq 3$ proved previously in [1]. On the other hand, in those cases where parabolicity is allowed, (e.g., when we consider Self-shrinkers $X_0 : \Sigma^n \to \mathbb{R}^{n+p}$), we have drawn some conclusions about its geometry.

References

[1] S. Markvorsen and V. Palmer, *Transience and capacity of minimal submanifolds*, GAFA, Geometric and Functional Analysis **13** (2003), 915–933.