Infinitesimal conformal transformations of the second degree in the Riemannian space of the second approximation

We consider a Riemannian space V_n, related to an arbitrary system of coordinates $\{x^1, x^2, \dots, x^n\}$. In the neighborhood of any fixed point $M_0(x_0^n)$, we construct the second approximation space $\tilde{V}_n^2(y^n; \tilde{g}_{ij}(y))$ by defining its metric tensor $\tilde{g}_{ij}(y)$ as:

$$\tilde{g}_{ij}(y) = g_{ij}\Big|_{\circ} + \frac{1}{3} R_{i\alpha\beta j} y^\alpha y^\beta \Big|_{\circ} \tag{1}$$

where $g_{ij}\Big|_{\circ} = g_{ij}(M_0), R_{i\alpha\beta j}\Big|_{\circ} = R_{i\alpha\beta j}(M_0)$.

In the space $\tilde{V}_n^2$, we consider infinitesimal conformal transformations of the second degree:

$$y'^h = y^h + \tilde{\xi}^h(y)\delta t \tag{2}$$

Definition. The transformations (2) are called transformations of the second degree, ([2]) if the displacement vector ξ ^ h (y) has the following form:

$$\tilde{\xi}^h(y) = a^h + a^h_{.l} y^l + a^h_{.l_1 l_2} y^{l_1 l_2} \tag{3}$$

where $a^h, a^h_{.l} y^l, a^h_{.l_1 l_2} y^{l_1 l_2}$.

We research generalized Killing equations ([1]):

$$\nabla_{(i\xi j)=} \psi(y) \tilde{g}_{ij}, \tag{4}$$

We get the following results.

Theorem 1. In the space of the second approximation $\tilde{V}_n^2$ for a Riemannian space V_n of nonzero constant curvature K, there exist infinitesimal conformal transformations of the second degree with the displacement vector $\tilde{\xi}^h(y)$ of the form:

$$\tilde{\xi}^h(y) = a^h + a^h_{.l} y^l - \frac{K}{3} a_l y^l y^h \tag{5}$$

where $a^h_{.l}$ satisfies the following condition:

$$a^\alpha_{.(i} \underset{\circ}{g}_{j)\alpha} = 0,$$

where $a^h_{.}$ is an arbitrary constant.

Theorem 2. The second-approximation space $\tilde{V}_n^2$ for a space of nonzero curvature admits a Lie group $G_2$of infinitesimal conformal transformations of the second degree of order r, where $r = \frac{n(n+1)}{2}$. The structure of this group is founded.

References

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