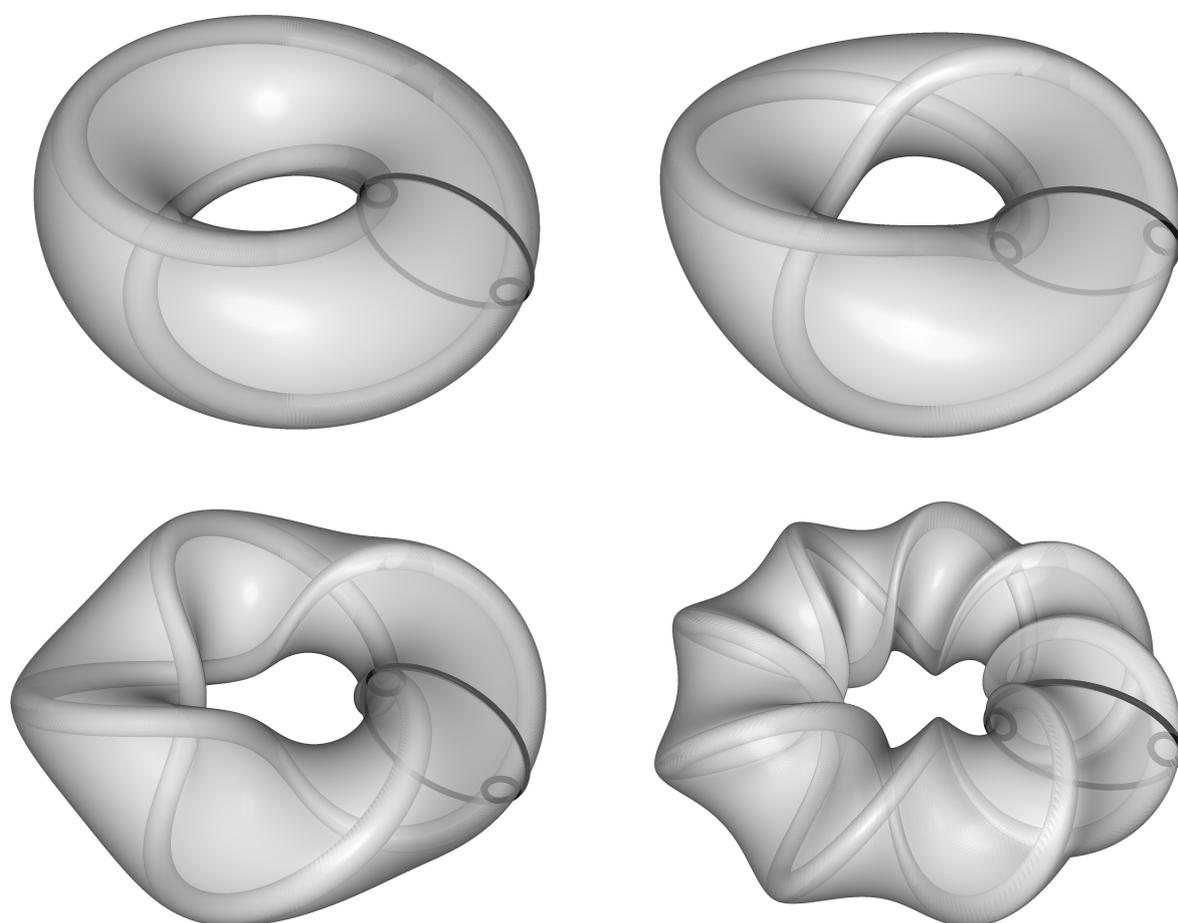


Differential Geometry 2017
Banach Conference center, Będlewo, Poland

June 19-23, 2017



Wendy Goemans, Barbara Opozda and Udo Simon

Program

Week overview:

Time slot	Monday 19th June 2017	Tuesday 20th June 2017	Wednesday 21st June 2017	Thursday 22nd June 2017	Friday 23rd June 2017
08.00-08.45	Breakfast - Registration	Breakfast	Breakfast	Breakfast	Breakfast
08.45-09.00	Opening ceremony				
09.00-09.50	Plenary (Derdzinski)	Plenary (Vrancken)	Plenary (Hertrich-Jeromin)	Plenary (Naveira)	40' (Albuje)
10.00-10.40	40' (Hall)	40' (Walczak)	40' (Munteanu)	40' (Palmer)	40' (Deszcz)
10.45-11.15	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11.15-11.55	40' parall. (Pok. - Cab.)	40' parall. (Tur. - Udr.)	40' parall. (Bej. - Csikós)	40' parall. (Rak. - Kırık)	Plenary (Tenenblat)
12.00-12.25	25' parall. (Krut. - Al.)	25' parall. (Şen - Woike)	25' parall. (Ew.-Kr. - Nis.)	25' parall. (Tad. - Sech.)	Closing
12.30-14.30	Lunch	Lunch	Lunch	Lunch	Lunch
14.30-14.55	25' parall. (Dr. - Kum.)	25' parall. (Gül. - Szcz.)	25' parall. (Bel. - Boy.)	25' parall. (Or. - N.-S.)	
15.00-15.25	25' parall. (Has. - Sr.)	25' parall. (Yun - Kell.)	25' parall. (V.-R. - Rob.)	25' parall. (Manfio - *)	Excursion to
15.30-15.55	25' parall. (Sch. - Tü.)	25' parall. (Hw. - C.-In.)	25' parall. (Szancer - *)	25' parall. (San.-L. - *)	Rogalin palace,
16.00-16.30	Coffee Break	Coffee Break	Coffee Break	Coffee Break	museum
16.30-17.10	40' (Cánovas)	40' (Stepanov)	40' (Yıldırım)	40' (Rovenski)	and park
17.15-18.05	Plenary (Carriazo)	Poster session	40' (Dušek)	Plenary (Simon)	
18.15-20.00	Dinner (Banquet)	Dinner	Dinner (Grill party)	Dinner	Dinner

Monday 19th June 2017

Time slot	19th June 2017	Speaker A	Title A	Speaker B	Title B
08.00-08.45	Registration				
08.45-09.00	Opening ceremony				
09.00-09.50	Plenary	DERDZINSKI	Compact Riemannian four-manifolds with harmonic curvature		
10.00-10.40	40'	HALL	Curvature and 4-dimensional Manifolds		
10.45-11.15	Coffee Break				
11.15-11.55	40' parallel	POKAS	Infinitesimal conformal transformations of the second degree in ...	CABALLERO	On the surfaces with the same mean curvature in the Euclidean 3-space and the Lorentz-Minkowski 3-space
12.00-12.25	25' parallel	KRUTOGOLOVA	... the Riemannian space of the second approximation	ALARCON	Geometric properties of spacelike hypersurfaces in the Lorentz-Minkowski space with the same Riemannian and Lorentzian mean curvature
12.30-14.30	Lunch				
14.30-14.55	25' parallel	DRACH	Blaschke's rolling ball theorem in Riemannian manifolds of bounded curvature	KUMAR	On geometry of real half lightlike submanifolds of indefinite Kaehler manifolds with a quarter symmetric metric connection
15.00-15.25	25' parallel	HASEGAWA	An inclusive immersion into a quaternionic manifold and its invariants	SROKA	Integrable complex structures on some 6-dimensional Lie algebras
15.30-15.55	25' parallel	SCHEUER	Harnack inequalities for evolving hypersurfaces	TURKOGLU	Special Transformations on Weyl Manifold with a special metric connection
16.00-16.30	Coffee Break				
16.30-17.10	40'	CANOVAS	Trapped submanifolds in de Sitter space		
17.15-18.05	Plenary	CARRIAZO	Slant submanifolds in semi-Riemannian manifolds		
18.15-20.00	Dinner (Banquet)				

Tuesday 20th June 2017

Time slot	20th June 2017	Speaker A	Title A	Speaker B	Title B
09.00-09.50	Plenary	VRANCKEN	Lagrangian submanifolds in the homogeneous nearly Kähler $S^3 \times S^3$ related to classical differential equations		
10.00-10.40	40'	WALCZAK	Dynamics of Darboux curves on surfaces		
10.45-11.15	Coffee Break				
11.15-11.55	40' parallel	TURGAY	Biconservative submanifolds with higher codimension in Riemannian space-forms	UDRISTE	Riemann flows and solitons
12.00-12.25	25' parallel	SEN	On the biconservative surfaces in Euclidean spaces	WOIKE	Symplectically fat bundles
12.30-14.30	Lunch				
14.30-14.55	25' parallel	GULER	Some characterizations of generalized Ricci solitons	SZEPKOWSKA	On Riemannian fat associated bundles
15.00-15.25	25' parallel	YUN	Rigidity of ricci solitons with weakly harmonic Weyl tensors	KELLECI	Geometrical properties of surfaces endowed with a canonical principal direction in 3-dimensional Minkowski space
15.30-15.55	25' parallel	HWANG	Besse conjecture with the Weyl tensor conditions	CASTRO-INFANTES	Complete dense minimal surfaces in any domain of \mathbb{R}^n
16.00-16.30	Coffee Break				
16.30-17.10	40'	STEPANOV	On infinitesimal harmonic transformations of complete Riemannian manifolds		
17.15-18.05	Poster session				
18.15-20.00	Dinner				

Wednesday 21st June 2017

Time slot	21st June 2017	Speaker A	Title A	Speaker B	Title B
09.00-09.50	Plenary	HERTRICH- JEROMIN	Transformations		
10.00-10.40	40'	MUNTEANU	New results on magnetic curves		
10.45-11.15	Coffee Break				
11.15-11.55	40' parallel	BEJAN	Einstein hypersurfaces of the cotangent bundle	CSIKOS	Harmonic Manifolds and Tubes
12.00-12.25	25' parallel	EWERT- KRZEMIENIEWSKI	On some classes of immersions into tangent bundle	NISTOR	Periodic magnetic curves on the 3-torus
12.30-14.30	Lunch				
14.30-14.55	25' parallel	BELOVA	Special form of geodesics and almost geodesics curves	BOYOM	The moduli space of statistical models
15.00-15.25	25' parallel	VALLE-REGEUIRO	Quasi-Einstein metrics and affine structures	ROBASZEWSKA	Bäcklund theorem for surfaces in the Galilean space G_3
15.30-15.55	25' parallel	SZANCER	Affine hyperspheres with a \tilde{J} -tangent Blaschke field		
16.00-16.30	Coffee Break				
16.30-17.10	40'	YILDIRIM	$\delta^\sharp(2, 2)$ -ideal hypersurfaces of dimension 5 in centroaffine differential geometry		
17.15-17.55	40'	DUSEK	Homogeneous geodesics in homogeneous Finsler spaces		
18.00-20.00	Dinner (Grill party)				

Thursday 22nd June 2017

Time slot	22nd June 2017	Speaker A	Title A	Speaker B	Title B
09.00-09.50	Plenary	NAVEIRA	Seven dimensional Bianchi-Cartan-Vranceanu spaces		
10.00-10.40	40'	PALMER	Some potential analysis on homothetic solutions to Mean Curvature Flow and Inverse Mean Curvature Flow		
10.45-11.15	Coffee Break				
11.15-11.55	40' parallel	RAKIC	Osserman manifolds and duality principle	KIRIK	Some results on recurrence structure of real and complex bivectors in space-times
12.00-12.25	25' parallel	TADANO	Some Cheeger-Gromov-Taylor Type Compactness Theorems for Ricci Solitons	SECHKIN	Topology of dynamics of a non-homogeneous rotationally symmetric ellipsoid on a smooth plane
12.30-14.30	Lunch				
14.30-14.55	25' parallel	ORTIZ	Stability and eigenvalue estimates for CMC surfaces in warped products	NIK.-SIM.	Homogeneity of Lorentzian three-manifolds with recurrent curvature
15.00-15.25	25' parallel	MANFIO	On a class of submanifolds in $S^n \times \mathbb{R}$ and $\mathbb{H}^n \times \mathbb{R}$		
15.30-15.55	25' parallel	SANMARTIN-LOPEZ	Isospectral submanifolds in non-compact symmetric spaces		
16.00-16.30	Coffee Break				
16.30-17.10	40'	ROVENSKI	Variational problems for a manifold equipped with a distribution		
17.15-18.05	Plenary	SIMON	Compact Ricci solitons in dimension two		
18.15-20.00	Dinner				

Friday 23rd June 2017

Time slot	23rd June 2017	Speaker A	Title A
09.00-09.40	40'	ALBUJER	A convexity result for constant mean curvature spacelike graphs in the Lorentz-Minkowski space
09.45-10.25	40'	DESZCZ	Curvature properties of some warped product manifolds
10.30-11.00	Coffee Break		
11.00-11.50	Plenary	TENENBLAT	On Dupin hypersurfaces with constant Laguerre curvature
12.00-13.25	Closing & Lunch		
13.30-18.55	Excursion to Rogalin palace, museum and park		
19.00-20.00	Dinner		

Postersession

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ALARCON, ALBUJER & CABALLERO	Geometric properties of spacelike hypersurfaces in the Lorentz-Minkowski space with the same Riemannian and Lorentzian mean curvature
ALEKSANDROVA	On conformal Killing and harmonic forms on Riemannian symmetric spaces
BOROWKA	Swann bundles, Armstrong cones and quaternionic Feix–Kaledin construction
DESZCZ & GLOGOWSKA	Curvature properties of some warped product manifolds
GOEMANS	Double rotational surfaces in Euclidean and Lorentz-Minkowski 4-space
KAPKA	An algebraic characterization of smooth vector fields using some universal property of $(\Omega^1(M), d)$
RASHEED	Riemann flows and solitons
TADANO	Diameter Bounds, Gap Theorems, and Hitchin-Thorpe Inequalities for Compact Quasi-Einstein Manifolds
WOZKNIAKOWSKA	The examples of non-Keller mappings
ZAWADSKI	Metrics and connections critical for the total mixed scalar curvature of a distribution

Abstracts

Geometric properties of spacelike hypersurfaces in the Lorentz-Minkowski space with the same Riemannian and Lorentzian mean curvature – Eva M. Alarcón

Spacelike hypersurfaces in the Lorentz-Minkowski space \mathbb{L}^{n+1} can be endowed with two Riemannian metrics, the one inherited from \mathbb{L}^{n+1} and the one induced by the Euclidean metric of \mathbb{R}^{n+1} . As a direct consequence of the classical theorems of Bernstein and Calabi-Bernstein, and of the generalization of the last one to arbitrary dimension, we can deduce that the only entire graphs in \mathbb{L}^{n+1} that are simultaneously minimal and maximal are the spacelike hyperplanes. Using a theorem of Heinz, Chern and Flanders, we can extend this result to entire spacelike graphs with the same constant mean curvature functions H_R and H_L .

We consider the general case of spacelike hypersurfaces with the same Riemannian and Lorentzian mean curvature functions not necessarily constant, and we study some of their geometric properties. Specifically, we prove that a spacelike hypersurface in \mathbb{L}^{n+1} such that $H_R = H_L$ does not have any elliptic points. As an application of this result jointly with a well-known result by Osserman about the non-existence of elliptic points for a certain class of compact hypersurfaces in \mathbb{R}^{n+1} , we give some interesting consequences about the geometry of such hypersurfaces.

This is a joint work with Alma L. Albuja and Magdalena Caballero.

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A convexity result for constant mean curvature spacelike graphs in the Lorentz-Minkowski space – Alma L. Albuja, Magdalena Caballero and Rafael López

A surface in the 3-dimensional Lorentz-Minkowski space \mathbb{L}^3 is said to be spacelike if its induced metric from \mathbb{L}^3 is Riemannian. In this contribution we consider compact spacelike surfaces, with non-zero constant mean curvature, immersed in \mathbb{L}^3 , and with (necessarily) non-empty, smooth and convex boundary. In this context, we ask about the influence of the geometry of the boundary on the surface. In particular, we prove that if the boundary is a planar curve, with the added property that it intersects any branch of any hyperbola at at most five points, then our surface is strictly convex. Let us observe that, under such assumptions, the boundary is always convex. Ellipses are a particular case of such curves.

The proof of our result follows the ideas of Chen and Huang [3], which were inspired by a previous argument by Alexandrov [2].

Finally, we also present an example that shows that, in general, the convexity of the boundary is not inherited by the surface. Therefore, we cannot omit the assumption on the minimum number of intersection points of our curve with a branch of a hyperbola.

References

- [1] A. L. Albuja, M. Caballero and R. López. *Convexity of the solutions to the constant mean curvature spacelike surface equation in the Lorentz-Minkowski space*. J. Differential Equations 258 (2015), 2364-2374.
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On conformal Killing and harmonic forms on Riemannian symmetric spaces – Irina Alexandrova and Sergey Stepanov

Conformal Killing forms have been defined on Riemannian manifolds more than forty-five years ago by Tachibana (see [1]) as a natural generalization of conformal Killing vector fields. Surveys of the publications on these forms can be found in the introduction to our last paper [2].

A Riemannian globally symmetric space of *non-compact type* (M, g) is complete and also (M, g) has a nonpositive sectional curvature. We also know that a Riemannian symmetric space has nonpositive (resp. non-negative) curvature operator if and only if it has nonpositive (resp. non-negative) sectional curvature (see [3]). Note that symmetric spaces of non-compact type are non-compact. After the above remarks, the assertion of the following theorem becomes obvious.

Theorem 1. *A globally symmetric space of non-compact type (M, g) with infinite volume $\text{Vol}_g(M)$ does not admit a nonzero conformal Killing L^2 -form.*

It is well known that a Riemannian globally symmetric space of compact type (M, g) is compact and also (M, g) has a nonpositive sectional curvature. Then the following theorem holds.

Theorem 2. *A globally symmetric space of compact type (M, g) does not admit a non-parallel harmonic form.*

Our work was supported by RBRF grant 16-01-00053-a (Russia).

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- [1] Tachibana S., On conformal Killing tensor in a Riemannian space, Tohoku Math. Journal 21 (1969) 56-64.
- [2] Alexandrova I.A., Mikes J., Stepanov S.E., Tsyganok I.I., Conformal Killing forms on totally umbilical submanifolds, J. Math. Sci. (NY) 217 (2016), no. 5, 525-539.
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Einstein hypersurfaces of the cotangent bundle – Cornelia-Livia Bejan

Let M be a manifold endowed with a symmetric connection. On its cotangent bundle, the Riemann extension (introduced by Patterson and Walker) was generalized by Kowalski and Sekizawa to the natural Riemann extension, which is a semi-Riemannian structure of neutral signature. On the total space of the cotangent bundle, we construct and study a family of hypersurfaces on which the natural Riemann extension is semi-Riemannian, i.e. non-degenerate. We prove that these hypersurfaces are Einstein.

Special form of geodesics and almost geodesics curves – Olga Belova

I determine in the n -dimensional real space the form of curves C for which also any image under an $(n-1)$ -dimensional algebraic torus is a geodesic or an almost geodesic with respect to an affine connections with constant coefficients and calculate explicitly the components of the connections. The geodesics and almost geodesics play an important role in differential geometry the explicit calculation of the form of curves C which are geodesics or almost geodesics with respect to an affine connection is not achievable even in the case if the components of the connection are constant. But we can do it if we moreover suppose that with C also all images of C under a real $(n-1)$ -dimensional algebraic torus are also geodesics, respectively almost geodesics. This implies that the determination of C becomes an algebraic problem. I considered a curve C homeomorphic to R which is a closed subset of n -dimensional real space and has the special form.

Swann bundles, Armstrong cones and quaternionic Feix-Kaledin construction – Aleksandra Borówka

B. Feix and D. Kaledin independently showed that there exists a hypercomplex structure on a neighbourhood of the zero section of the tangent bundle of a complex manifold with a real analytic connection with curvature of type $(1, 1)$. With D. Calderbank, we generalized this construction by allowing a twist by a line bundle with a connection and placing it in the framework of projective geometry: we showed that a neighbourhood of the zero section of any twisted tangent bundle of a c -projective manifold admits a quaternionic structure. Moreover, we completely characterize quaternionic manifolds that can be obtained in this way.

In this talk we will outline this construction and discuss its relations to twisted Armstrong cones and Swann bundles.

The moduli space of statistical models – Michel Nguiffo Boyom

The classical theory of statistical models deals with open subsets of Euclidean space. This yields the lack of Geometry. The aim of my talk is to revisit the foundation of the information Geometry. I will overview the subject and address the moduli space of isomorphism class of statistical models.

On the surfaces with the same mean curvature in the Euclidean 3-space and the Lorentz-Minkowski 3-space – Magdalena Caballero

Spacelike surfaces in the Lorentz-Minkowski 3-dimensional space \mathbb{L}^3 can be endowed with another Riemannian metric, the one induced by the Euclidean space \mathbb{R}^3 . Those surfaces are locally the graph of a smooth function $u(x, y)$ satisfying $|Du| < 1$. If in addition they have the same mean curvature with respect to both metrics, they are the solutions to a certain partial differential equation, the $H_R = H_L$ surface equation.

It is well known that the only surfaces that are simultaneously minimal in \mathbb{R}^3 and maximal in \mathbb{L}^3 are open pieces of helicoids and of spacelike planes, [3].

In this talk we consider the general case of spacelike surfaces with the same mean curvature with respect to both metrics. We prove the existence of examples with non-zero mean curvature, we show geometric properties of these surfaces and we study the $H_R = H_L$ equation.

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Trapped submanifolds in de Sitter space – Verónica L. Cánovas

The concept of trapped surfaces was originally formulated by Penrose for the case of 2-dimensional spacelike surfaces in 4-dimensional spacetimes in terms of the signs or the vanishing of the so-called null expansions. This is obviously related to the causal orientation of the mean curvature vector of the surface, which provides a better and powerful characterization of the trapped surfaces and allows the generalization of this concept to codimension two spacelike submanifolds of arbitrary dimension n . In this sense, an n -dimensional spacelike submanifold Σ of an $(n + 2)$ -dimensional spacetime is said to be *future trapped* if its mean curvature vector field \vec{H} is timelike and future-pointing everywhere on Σ , and similarly for *past trapped*. If \vec{H} is lightlike (or null) and future-pointing everywhere on Σ then the submanifold is said to be *marginally future trapped*, and similarly for *marginally past trapped*. Finally, if \vec{H} is causal and future-pointing everywhere,

the submanifold is said to be *weakly future trapped*, and similarly for *weakly past trapped*. The extreme case $\vec{H} = \vec{0}$ on Σ corresponds to a minimal submanifold.

In this lecture we consider codimension two compact marginally trapped submanifolds in the light cone of de Sitter space. In particular, we show that they are conformally diffeomorphic to the round sphere and, as an application of the solution of the Yamabe problem on the round sphere, we derive a classification result for such submanifolds. We also fully describe the codimension two compact marginally trapped submanifolds contained into the past infinite of the steady state space.

This is part of our work in progress with Luis J. Alías (from Murcia) and Marco Rigoli (from Milano).

Slant submanifolds in semi-Riemannian manifolds – Alfonso Carriazo

Slant submanifolds were defined by B.-Y. Chen as a natural generalization of both complex and totally real submanifolds of an almost Hermitian manifold. In this talk, we will review the definitions of slant submanifolds in different ambient spaces, with special attention to the non-Riemannian ones. We will offer their main properties and examples, as well as some recently obtained new results.

Complete dense minimal surfaces in any domain of \mathbb{R}^n – Ildefonso Castro-Infantes

Given an open Riemann surface M and an integer $n \geq 3$, we prove on [1] that the set of complete conformal minimal immersions $M \rightarrow \mathbb{R}^n$ with $\overline{X(M)} = \mathbb{R}^n$ forms a dense subset in the space of all conformal minimal immersions $M \rightarrow \mathbb{R}^n$ endowed with the compact-open topology. Furthermore, we show that given a domain in \mathbb{R}^n we may find a complete minimal surface which is dense on it and has arbitrary orientable topology (possibly infinite); we also provide such surfaces whose complex structure is any given bordered Riemann surface.

Analogous results for non-orientable minimal surfaces in \mathbb{R}^n ($n \geq 3$), complex curves in \mathbb{C}^n ($n \geq 2$), holomorphic null curves in \mathbb{C}^n ($n \geq 3$), and holomorphic Legendrian curves in \mathbb{C}^{2n+1} ($n \in \mathbb{N}$) may be proved adapting the used techniques.

References

[1] A. Alarcón, I. Castro-Infantes. *Complete minimal surfaces densely lying in arbitrary domains of \mathbb{R}^n* arXiv:1611.05029.

Harmonic Manifolds and Tubes – Balázs Csikós and Márton Horváth

It was shown in a preceding paper [1] that in a connected locally harmonic manifold, the volume of a tube of small radius about a regularly parameterized simple arc depends only on the length of the arc and the radius. In this talk, we show that this property characterizes harmonic manifolds even if it is assumed only for tubes about geodesic segments. As a consequence, we obtain similar characterizations of harmonic manifolds in terms of the total mean curvature and the total scalar curvature of tubular hypersurfaces about curves. We find simple formulae expressing the volume, total mean curvature, and total scalar curvature of tubular hypersurfaces about a curve in a harmonic manifold as a function of the volume density function. The talk is based on the paper [2].

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[2] B. Csikós, M. Horváth, *Harmonic manifolds and tubes*, preprint: arXiv:1705.00311 [math.DG] (2017)

The research was supported by the Hungarian Scientific Research Fund OTKA K112703.

Compact Riemannian four-manifolds with harmonic curvature – Andrzej Derdzinski

We describe a step towards a classification of compact four-dimensional Riemannian manifolds whose curvature tensor R is harmonic as a 2-form valued in 2-forms or - equivalently - whose Ricci tensor satisfies the Codazzi equation. The known examples of such manifolds form five (non-disjoint) classes, in which the metric is, respectively, Einstein; conformally flat with constant scalar curvature; locally reducible (of types $1 + 3$ or $2 + 2$); and a $2 + 2$ warped product. This talk presents work in progress, joint with Paolo Piccione, showing how the question of classifying compact four-manifolds with harmonic curvature that lie outside of the five classes named above is reduced to a problem in real algebraic geometry.

Curvature properties of some warped product manifolds – Ryszard Deszcz, Małgorzata Głogowska, Marian Hotłoś and Jan Jełowicki

In this poster we will present curvature properties of pseudosymmetry type of the warped product manifolds $\overline{M} \times_F \tilde{N}$, with p -dimensional base manifold $(\overline{M}, \overline{g})$, $p = 1, 2$, $(n - p)$ -dimensional fiber (\tilde{N}, \tilde{g}) , $n \geq 4$, and the warping function F . These conditions are formed from the metric tensor g , the Riemann-Christoffel curvature tensor R , the Ricci tensor S and the Weyl conformal curvature tensor C of the considered manifolds. We will present results obtained in [1]-[8]. For instance, if $p = 1$ and the fiber (\tilde{N}, \tilde{g}) , $n \geq 5$, is an Einstein manifold, not of constant curvature, then $\overline{M} \times_F \tilde{N}$ is a quasi-Einstein non-pseudosymmetric Ricci-pseudosymmetric manifold and its difference tensor $R \cdot C - C \cdot R$ is a linear combination of the Tachibana tensors $Q(S, R)$ and $Q(g, R)$. If $p = 2$ and the fiber (\tilde{N}, \tilde{g}) , $n \geq 5$, is a semi-Riemannian space of constant curvature then the $(0, 6)$ -tensors $R \cdot R - Q(S, R)$ and $C \cdot C$ of such warped products are proportional to the Tachibana tensor $Q(g, C)$, and the tensor C is a linear combination of some Kulkarni-Nomizu products formed from the tensors g and S .

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Hypersurfaces in space forms satisfying some curvature conditions – Ryszard Deszcz and Małgorzata Głogowska

In this talk we will present curvature properties of pseudosymmetry type of hypersurfaces isometrically immersed in $(n + 1)$ -dimensional spaces of constant curvature, $n \geq 4$, having at every point at most three distinct principal curvatures. Under some additional assumptions, such hypersurfaces are pseudosymmetric, Ricci-pseudosymmetric, manifolds with pseudosymmetric Weyl tensor, or satisfy other conditions of this kind (see [1]-[9] and references therein). For instance, type number two hypersurfaces are pseudosymmetric manifolds of constant type, non-conformally flat and non-Einstein hypersurfaces with two distinct principal curvatures are Roter type manifolds, the Cartan hypersurfaces of dimension 6, 12 or 24 are non-pseudosymmetric Ricci-pseudosymmetric manifolds of constant type, and 2-quasi-umbilical hypersurfaces are manifolds with pseudosymmetric Weyl tensor.

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Blaschke's rolling ball theorem in Riemannian manifolds of bounded curvature – Kostiantyn Drach

In the talk we will show how to extend classical Blaschke's rolling ball theorem to the case of strictly convex domains lying in complete Riemannian manifolds of bounded sectional curvature. As the principal tools for such an extension, we will employ several comparison-type results, including celebrated Toponogov's triangle comparison theorem.

Homogeneous geodesics in homogeneous Finsler spaces – Zdeněk Dušek

The present talk is based on the preprint [2], in which homogeneous geodesics in Finsler homogeneous spaces are studied using the affine method, which was developed in papers [1] and [3]. We prove that any homogeneous Finsler space of odd dimension admits a homogeneous geodesic through any point. This statement was proved incorrectly in [5], using the algebraic method developed for the Riemannian situation in [4]. We further prove that any homogeneous Berwald space or homogeneous reversible Finsler space admits a homogeneous geodesic through any point.

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On some classes of immersions into tangent bundle – Stanisław Ewert-Krzemieniewski

The isometric immersion $f : (M, g) \rightarrow (N, \bar{g})$ of Riemannian manifolds gives rise in a natural way to immersions $\tilde{f} : TM \rightarrow TN$ or $\tilde{f} : T^\perp M \rightarrow TN$, where TN is equipped with non-degenerate \bar{g} -natural metric \bar{G} . We propose investigation of such immersions generated by vector fields either tangent to M or normal to M .

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Double rotational surfaces in Euclidean and Lorentz-Minkowski 4-space – Wendy Goemans

One of the most basic examples of surfaces in 3-dimensional Euclidean space is a rotational surface or a surface of revolution, that is, a surface which is the trace of a planar curve that is rotated about an axis in its supporting plane. Its simple construction makes that it is appealing to geometers, but also is open to alteration. One possible generalization is to subject a planar curve to two simultaneous rotations. The resulting surface is called a twisted surface and studied in e.g. [2] (see also the references therein). Another possibility is to extend the concept of a rotational surface to higher dimensional ambient spaces, see e.g. [3, 4]. Combining these two points of view leads to the construction of a double rotational surface in 4-space: perform on a planar curve in 4-space two simultaneous rotations, possibly at different rotation speeds.

In this contribution I want to present double rotational surfaces in 4-space and advertise them as possible research subjects, possible (counter-) examples and inspirational objects for further research. Also I want to comment on a partial result about these surfaces, namely, on flat double rotational surfaces and their relation with Clelia curves, see [1]. The ambient 4-space will be Euclidean or Lorentz-Minkowski 4-space.

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Some characterizations of generalized Ricci solitons – Sinem Güler and Sezgin Altay Demirbağ

The importance of Ricci solitons comes from the fact that they are corresponding to self-similar solutions of the Ricci flow and at the same time they are natural generalizations of Einstein metrics. Some generalizations like gradient Ricci solitons [1], quasi Einstein manifolds [2] and generalized quasi Einstein manifolds [3] play important roles to classify the self-similar solutions of geometric flows and to describe the local structure of certain manifolds. In this talk, we focus on another generalization which is generalized Ricci soliton [4] introduced as a class of overdetermined system of equations

$$\mathcal{L}_X g + 2\alpha X^\flat \otimes X^\flat - 2\beta \text{Ric} = 2\lambda g$$

on pseudo-Riemannian manifolds (M^n, g) for some vector field X and some real constants α , β and δ , where $\mathcal{L}_X g$ and X^\flat denote the Lie derivative of the metric g in the direction of X and the canonical 1-form associated to X , respectively. Briefly, the main objective of this work is to understand the relationship between certain vector fields and generalized Ricci solitons.

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Curvature and 4-dimensional Manifolds – Graham Hall

In this talk I will discuss the curvature structure of a 4-dimensional manifold with metric g , associated Levi-Civita connection ∇ , holonomy group Φ , curvature tensor $Riem$, sectional curvature function σ and Weyl conformal tensor C . I will concentrate on the case when g has neutral signature since the other signatures are known. A brief review will be given of the isometry group $O(2, 2)$ associated with g and the holonomy subalgebras of $o(2, 2)$. Some strong relationships between g , ∇ , Φ and σ and also between C and the conformal class of g will be briefly described, for example, that, except in some very special cases, σ uniquely determines g and C uniquely determines the conformal class of g . This will involve the introduction of the *curvature/Weyl map*. The rest of the talk will involve a classification of $Riem$ using the curvature map and will concentrate on the extent to which $Riem$ determines g and ∇ . A simple corollary concerning the symmetries of $Riem$ then follows.

An inclusive immersion into a quaternionic manifold and its invariants – Kazuyuki Hasegawa

We say that (M, Q) is a quaternionic manifold with the quaternionic structure Q if Q is a subbundle of $\text{End}(TM)$ with $\text{rank } Q = 3$ which is locally spanned by I_1, I_2, I_3 satisfying $I_1^2 = I_2^2 = I_3^2 = -id$, $I_1 I_2 = -I_2 I_1 = I_3$, and there exists a torsion free affine connection ∇ which preserves Q . Such a torsion free affine connection ∇ is called a quaternionic connection. Note that the quaternionic connection is not unique. Our interests are objects and properties which are independent of the choice of quaternionic connections. Let Σ be an oriented surface and $f : \Sigma \rightarrow M$ an immersion. If $f_{*x}(T_x \Sigma)$ is contained in a real 4-dimensional quaternionic subspace of $T_{f(x)}M$ for each point $x \in \Sigma$, then $f : \Sigma \rightarrow M$ is called an inclusive immersion or Σ is called

an inclusive surface. If f is an inclusive immersion from an oriented surface, then there exists a unique lift $I_1 : \Sigma \rightarrow \mathcal{Z}$ such that $I_1(x)$ preserves $f_*(T_x\Sigma)$ at each point $x \in \Sigma$ and the induced complex structure I on Σ is compatible with the given orientation of Σ , where \mathcal{Z} is the twistor space of M . See [AM]. The map I_1 is called the natural twistor lift of f .

We introduce a quaternionic invariant for an inclusive immersion into a quaternionic manifold, which will be denoted by \mathcal{W}_Q in this talk. When $M = \mathbf{HP}^1(\cong S^4)$, the functional \mathcal{W}_Q coincides with the conformal Willmore functional. Therefore we may consider that \mathcal{W}_Q is a candidate for a quaternionic object of the Willmore functional, because a four-dimensional quaternionic manifold is simply an oriented manifold with a conformal structure. The lower bound of this invariant is given by topological invariant and the equality case can be characterized in terms of the natural twistor lift. When $M = \mathbf{HP}^1$ and the natural twistor lift $I_1 : \Sigma \rightarrow \mathbf{CP}^{2n+1}$ is holomorphic, we obtain a relation between the quaternionic invariant and the degree of the image of the natural twistor lift as an algebraic curve. Moreover the first variation formula for the invariant is obtained. As an application of the formula, if the natural twistor lift is a harmonic section, then the surface is a stationary point under any variations such that the induced complex structures do not vary. This is a quaternionic version and a generalized result in [BC] for a constrained Willmore surface.

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Transformations – Udo Hertrich-Jeromin

Transformations of curves and surfaces may be used to generate new shapes from known ones - while preserving key properties of the original shape. On the other hand, transformations and their permutability relations may serve to obtain integrable discretizations.

We shall discuss the sphere geometric zoo of transformations of curves and surfaces (i.e., by Ribaucour, Darboux, Christoffel, etc) and how they play a role in discretization and shape generation.

Besse conjecture with the Weyl tensor conditions – Seungsu Hwang

On a compact n -dimensional manifold, it is well known that a critical metric of the total scalar curvature, restricted to the space of metrics with unit volume is Einstein. It has been conjectured that a critical metric of the total scalar curvature, restricted to the space of metrics with constant scalar curvature of unit volume, will be Einstein. This conjecture, proposed in 1987 by Besse, has not been resolved. In this talk, we prove the Besse conjecture with various vanishing conditions on the Weyl tensor.

An algebraic characterization of smooth vector fields using some universal property of $(\Omega^1(M), d)$ – Sławomir Kapka

We present a new approach to the well known algebraic characterization of smooth vector fields. We show that isomorphism $\text{Der}_{\mathbb{R}}(C^\infty(M)) \cong \mathfrak{X}(M)$ of $C^\infty(M)$ -modules can be recovered using some universal property of $(\Omega^1(M), d)$. More precisely, we use the fact that $(\Omega^1(M), d)$ is the universal derivation in the category of geometric $C^\infty(M)$ -modules.

We also shed new light on this universality of $(\Omega^1(M), d)$ by showing that it is a consequence of Hadamard's Lemma. Some simple ideas behind considering geometric $C^\infty(M)$ -modules we cover as well.

Geometrical properties of surfaces endowed with a canonical principal direction in 3-dimensional Minkowski space – Alev Kelleci

Given a vector field X in a Riemannian manifold N , a hypersurface M of N is said to be endowed with a canonical principal direction relative to X if the projection of X onto the tangent bundle of M gives a principal direction, [4].

It turns out that when N is a product space $\tilde{N} \times \mathbb{R}$ some interesting geometrical properties of hypersurfaces endowed with a canonical principal direction relative to X occur if X is chosen to be the unit vector field tangent to the second factor (See for example [1, 2, 3, 6, 8]). On the other hand, some particular cases of this problem were studied in [5, 7], where the ambient space N is (pseudo-)Euclidean and X is a fixed direction.

In this talk, we will focus on surfaces in Minkowski 3-space \mathbb{E}_1^3 after we will present a survey of recent results on surfaces having a canonical principal direction relative to X . In particular, we present some new classification results of these surfaces in \mathbb{E}_1^3 when X is chosen to be a fixed direction.

2000 MSC Codes. Primary 53C50; Secondary 53C42

Keywords: Canonical principal direction, Minkowski 3-space.

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Some results on recurrence structure of real and complex bivectors in space-times – Bahar Kırık

The purpose of this study is to investigate the recurrence structure of bivectors (second order skew-symmetric tensors) on 4-dimensional manifolds having a metric of Lorentz signature. First of all, this structure is examined for real bivectors by analysing parallel (or scalable to parallel) or proper recurrence for such tensor fields. After that, this examination is extended to complex bivectors on these manifolds. Some remarks and theorems are given regarding the existence of a real, null, recurrent vector field. All these results are associated with the possible holonomy algebras for Lorentz manifolds. Conditions are expressed regarding the possibility that the manifold admits a properly recurrent bivector and a parallel bivector. A brief review is also done for second order symmetric, recurrent tensor fields for 4-dimensional manifolds admitting a metric of Lorentz, neutral or positive definite signature.

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On Geometry of Real Half Lightlike Submanifolds of Indefinite Kaehler Manifolds with a Quarter Symmetric Metric Connection – Rakesh Kumar

We study real half lightlike submanifolds of an indefinite Kaehler manifold with quarter symmetric metric connection. We find a necessary and sufficient condition for the screen distribution of a real half lightlike submanifold to be integrable. We obtain some characterization theorems for a real half lightlike submanifold to be a totally geodesic and a product manifold. We also find conditions for the null sectional curvature of a real half lightlike submanifold of an indefinite complex space form to be vanishes. Finally, we derive the expression for induced Ricci type tensor $R^{(0,2)}$ and obtain conditions for $R^{(0,2)}$ to be symmetric.

On a class of submanifolds in $S^n \times \mathbb{R}$ and $\mathbb{H}^n \times \mathbb{R}$ – Fernando Manfio

Given an isometric immersion $f : M^m \rightarrow \mathbb{Q}_\epsilon^n \times \mathbb{R}$, where \mathbb{Q}_ϵ^n denote either the unit sphere S^n or the hyperbolic space \mathbb{H}^n , according as $\epsilon = 1$ or $\epsilon = -1$, respectively, let ∂_t be a unit vector field tangent to the second factor. Then, a tangent vector field T on M^m and a normal vector field η along f are defined by

$$\partial_t = f_*T + \eta.$$

Let A denote the class of isometric immersions $f : M^m \rightarrow \mathbb{Q}_\epsilon^n \times \mathbb{R}$ with the property that T is an eigenvector of all shape operators of f . In this talk we will discuss some important subclasses of A , such as submanifolds with constant sectional curvature [3], rotational submanifolds [1], constant angle hypersurfaces [6] and, with more details, recent works about biconservative submanifolds with parallel mean curvature vector field [2], [4].

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New results on magnetic curves – Marian Ioan Munteanu

Magnetic curves represent the trajectories of the charged particles moving on a Riemannian manifold under the action of the magnetic fields. They are modeled by a second order differential equation, that is $\nabla_{\gamma'}\gamma' = \varphi\gamma'$, usually known as the Lorentz equation. Such curves are sometimes called also magnetic geodesics since the Lorentz equation generalizes the equation of geodesics under arc-length parametrization, namely, $\nabla_{\gamma'}\gamma' = 0$. In the last years, magnetic curves were studied in Kähler manifolds and Sasakian manifolds, respectively, since their fundamental 2-forms provide natural examples of magnetic fields.

In this talk we present our recent investigation on magnetic curves on the unit tangent bundle of a Riemannian manifold M . We write the equation of motion for arbitrary M . In the case when M is a space form $M(c)$, we prove that every contact magnetic curve is slant. If $c \neq 1$, a contact normal magnetic curve is slant if and only if it satisfies a conservation law. These results generalize the beautiful paper of Klingenberg and Sasaki published in 1975 about geodesics on the unit tangent bundle of the 2-sphere.

This presentation is based on the following paper:

J. Inoguchi and M. I. Munteanu: *Magnetic curves on tangent sphere bundles*, submitted.

Seven dimensional Bianchi-Cartan-Vranceanu spaces – A. Ferrández, Antonio Martínez Naveira and A. D. Tarrío

It is well known the differential geometry of 3-dimensional Bianchi, Cartan and Vranceanu (BCV) spaces W^3 . We give a natural seven dimensional generalization W^7 of those spaces and study some properties, such as the Levi-Civita connection, Ricci curvatures and Killing vector fields. We also show that W^3 is an immersed submanifold of W^7 .

Homogeneity of Lorentzian three-manifolds with recurrent curvature – Stana Nikčević Simić

k -Curvature homogeneous three-dimensional Walker metrics are described for $k = 0, 1, 2$. This allows a complete description of locally homogeneous three-dimensional Walker metrics, showing that there exist exactly three isometry classes of such manifolds. As an application one obtains a complete description of all locally homogeneous Lorentzian manifolds with recurrent curvature. Moreover, potential functions are constructed in all the locally homogeneous manifolds resulting in steady gradient Ricci and Cotton solitons.

Periodic magnetic curves on the 3-torus – Ana-Irina Nistor

We consider two magnetic fields on the 3-torus obtained from two different contact forms on the Euclidean 3-space. We study when their corresponding magnetic curves are closed. We find periodicity conditions involving the set of rational numbers. As the spectrum of closed geodesics

on the torus is analogue to the quantization of the energy levels in models of atoms, we called the obtained condition the quantization principle.

This presentation is based on the following paper:

M.I. Munteanu, A.I. Nistor, *On some closed magnetic curves on a 3-torus*, Math. Phys. Analysis Geometry 20 (2017) 2, art. 8.

Stability and eigenvalue estimates for CMC surfaces in warped products – Irene Ortiz

Constant mean curvature surfaces (CMC) are characterized as critical points of the area functional restricted to those variations which preserve certain volume function. For such critical points the stability is given by the Jacobi operator J , then a surface is said to be stable if the first eigenvalue associated to the mentioned operator is non negative.

Our aim is the search for estimates for the first eigenvalue of the Jacobi operator of compact CMC surfaces immersed into three-dimensional warped products. We also characterize the cases when the upper bound is reached. As an application, we derive some consequences for those surfaces that are stable, obtaining some classification results.

This is a joint work with Miguel A. Meroño.

Some potential analysis on homothetic solutions to Mean Curvature Flow and Inverse Mean Curvature Flow – Vicente Palmer

We study the two simplest and classical elliptic PDE equations, namely, Laplace's and Poisson's equation defined on Self-shrinkers and Self-expanders for the MCF and Solitons, (with positive or negative velocity constant), for the IMCF. A classical question in this field consists in to explore the existence of bounded and non-constant subharmonic functions defined in the (sub)-manifold, $X_0 : \Sigma^n \rightarrow \mathbb{R}^{n+p}$, (i.e., its parabolicity). We have proved that Self-expanders $X_0 : \Sigma^n \rightarrow \mathbb{R}^{n+p}$ for the MCF with $n \geq 3$ and Solitons $X_0 : \Sigma^n \rightarrow \mathbb{R}^{n+p}$ for the IMCF with negative velocity constant and $n \geq 2$ or positive velocity constant $c > \frac{1}{n-2}$ and $n \geq 3$ are non-parabolic.

If we consider, in some sense, the minimal submanifolds in Euclidean space as a "limit" case of Self-expanders, the non-parabolicity of Self-expanders is then coherent with non-parabolicity of minimal submanifolds $X : \Sigma^n \rightarrow \mathbb{R}^{n+p}$ with $n \geq 3$ proved previously in [1]. On the other hand, in those cases where parabolicity is allowed, (e.g., when we consider Self-shrinkers $X_0 : \Sigma^n \rightarrow \mathbb{R}^{n+p}$), we have drawn some conclusions about its geometry.

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Infinitesimal conformal transformations of the second degree in the Riemannian space of the second approximation – Sergey M. Pokas and Alina V. Krutoholova

We consider a Riemannian space V_n , related to an arbitrary system of coordinates $\{x^1, x^2, \dots, x^n\}$. In the neighborhood of any fixed point $M_0(x_0^n)$, we construct the second approximation space $\tilde{V}_n^2(y^n; \tilde{g}_{ij}(y))$ by defining its metric tensor $\tilde{g}_{ij}(y)$ as:

$$\tilde{g}_{ij}(y) = g_{ij} + \frac{1}{3} R_{i\alpha\beta j} y^\alpha y^\beta,$$

where $g_{ij} = g_{ij}(M_0)$, $R_{i\alpha\beta j} = R_{i\alpha\beta j}(M_0)$.

In the space \tilde{V}_n^2 , we consider infinitesimal conformal transformations of the second degree:

$$y'^h = y^h + \tilde{\xi}^h(y) \delta t \tag{1}$$

Definition. The transformations (1) are called transformations of the second degree, ([2]) if the displacement vector $\tilde{\xi}^h(y)$ has the following form:

$$\tilde{\xi}^h(y) = a^h + a_{.l}^h y^l + a_{.l_1 l_2}^h y^{l_1 l_2}$$

where $a^h, a_{.l}^h y^l, a_{.l_1 l_2}^h y^{l_1 l_2}$.

We research generalized Killing equations ([1]):

$$\nabla_{(i\xi_j)} = \psi(y)\tilde{g}_{ij},$$

We get the following results.

Theorem 1. In the space of the second approximation \tilde{V}_n^2 for a Riemannian space V_n of nonzero constant curvature K , there exist infinitesimal conformal transformations of the second degree with the displacement vector $\tilde{\xi}^h(y)$ of the form:

$$\tilde{\xi}^h(y) = a^h + a_{.l}^h y^l - \frac{K}{3} a_l y^l y^h$$

where a_l^h satisfies the following condition:

$$a_{.(i_0^g j)\alpha} = 0,$$

where a^h is an arbitrary constant.

Theorem 2. The second-approximation space \tilde{V}_n^2 for a space of nonzero curvature admits a Lie group G_2 of infinitesimal conformal transformations of the second degree of order r , where $r = \frac{n(n+1)}{2}$. The structure of this group is founded.

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Osserman manifolds and duality principle – Zoran Rakić

Let (M, g) be a pseudo-Riemannian manifold, with curvature tensor R . The Jacobi operator R_X is the symmetric endomorphism of $T_p M$ defined by $R_X(Y) = R(Y, X)X$. In Riemannian settings, if M is locally a rank-one symmetric space or if M is flat, then the local isometry group acts transitively on the unit sphere bundle SM and hence the eigenvalues of R_X are constant on SM . Osserman in the late eighties, wondered if the converse held; this question is usually known as the *Osserman conjecture*.

In the last twenty five years many authors have been studied problems which arising from the Osserman conjecture and its generalizations. In the first part of the lecture we will give an overview of Osserman type problems in the pseudo-Riemannian geometry. The second part is devoted to the equivalence of the Osserman pointwise condition and the duality principle. This part of the lecture consists the new results, which are obtained in collaboration with Yury Nikolayevsky and Vladica Andrejić.

Bäcklund theorem for surfaces in the Galilean space G_3 – Maria Robaszewska

The 3-dimensional Galilean geometry is the pair $(\mathbf{R}^3, \mathcal{G})$, where \mathcal{G} is the 6-parameter group of transformations which have the form

$$\begin{aligned}\bar{x} &= x + a, \\ \bar{y} &= bx + \cos \varphi y + \sin \varphi z + c, \\ z &= ex - \sin \varphi y + \cos \varphi z + f\end{aligned}$$

if we choose the coordinates x, y, z in a standard way, that is, Euclidean planes correspond to $x = \text{const}$. This geometry can be thought of as the geometry of classical kinematics in the Euclidean (y, z) plane with the time variable x .

The length $|v|$ of a vector $v = (X, Y, Z)$ is equal to $|X|$ if $X \neq 0$ and $\sqrt{Y^2 + Z^2}$ if $X = 0$. The vectors with $X = 0$ are called isotropic. The scalar product of isotropic vectors is defined as the restriction of the standard scalar product of \mathbf{R}^3 to the Euclidean plane. Having the scalar product we can measure the angle between two isotropic vectors.

In the local theory of surfaces in the Galilean space one considers only surfaces which have no Euclidean tangent planes. Such surfaces are called *admissible*. On an admissible surface one can define – up to sign – the Galilean normal vector field \mathbf{n} (isotropic, perpendicular to the unique isotropic tangent direction $\mathbf{R}\sigma$ and of unit length). By the Gauss formula with \mathbf{n} in the place of a transversal field one defines a linear connection ∇ and the second fundamental form of a surface. The *Gaussian curvature* K and the *mean curvature* H are also defined in the Galilean geometry. A surface is called minimal if $H \equiv 0$.

A non-flat Galilean connection ∇ always satisfies the condition $\dim \text{im } R = 1$. One can prove that every surface f with locally symmetric Blaschke connection satisfying the condition $\dim \text{im } R = 1$ is a surface of constant Gaussian curvature in the Galilean space G_3 . (If we use the standard coordinates in G_3 , then we have to compose f with some affine isomorphism of \mathbf{R}^3 .)

One can also prove some Galilean versions of the Bäcklund theorem. We assume that $f, \hat{f} : M \rightarrow G_3$ are non-degenerate admissible immersions such that $f_*(T_pM) \cap \hat{f}_*(T_pM) = \mathbf{R}(\hat{f}(p) - f(p))$ for every $p \in M$.

Theorem 1. *If $\hat{f} - f$ is everywhere non-isotropic, of constant (Galilean) length $|\hat{f} - f| = L$, and the angle α between the Galilean normal vector fields \mathbf{n} and $\hat{\mathbf{n}}$ is constant with $\sin \alpha \neq 0$, then both surfaces are of constant negative Galilean Gaussian curvature $K = \hat{K} = -\frac{\sin^2 \alpha}{L^2}$. The second fundamental forms h and \hat{h} are proportional.*

Theorem 2. *If $\mathbf{n} \parallel \hat{\mathbf{n}}$, then f and \hat{f} are minimal.*

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Variational problems for a manifold equipped with a distribution – Vladimir Rovenski

A distribution \mathcal{D} on a manifold M appears in various situations, e.g. tangent bundle of a foliation or kernel of a differential form. We discuss two curvature related functionals on the space of metrics on (M, \mathcal{D}) .

1. The mixed scalar curvature S_{mix} is the simplest invariant of a metric on (M, \mathcal{D}) . For a stably causal (e.g. globally hyperbolic) spacetime, which is naturally endowed with a codimension-one distribution, the total S_{mix} is an analog of Einstein-Hilbert action. We show that the Euler-Lagrange equations for any (M, \mathcal{D}) look like Einstein field equations with the new Ricci type curvature.

2. Given M^3 equipped with a plane field \mathcal{D} and a vector field T transverse to \mathcal{D} , we use 1-form ω such that $\mathcal{D} = \ker \omega$ and $\omega(T) = 1$ to construct a 3-form analogous to the Godbillon-Vey class of a foliation. For a metric g on M , we express this form in terms of geometry of \mathcal{D} and the curvature and torsion of its normal curves and derive Euler-Lagrange equations of associated action.

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Isospectral submanifolds in non-compact symmetric spaces – Víctor Sanmartín-López

Let M be a submanifold of \bar{M} with codimension greater or equal than two. In this work, we say that M is an isospectral submanifold if the spectrum of the shape operator S_ξ does not depend on the normal vector ξ . In particular, isospectral submanifolds are also examples of minimal or austere submanifolds.

According to [3], these kind of submanifolds appear in a natural way as the focal submanifolds of isoparametric families of hypersurfaces with constant principal curvatures in Riemannian manifolds. In this sense, isospectral submanifolds seem the suitable candidates for constructing isoparametric families of hypersurfaces.

In this talk, we will present a method which allows us to construct isospectral submanifolds in the context of symmetric spaces G/K of non-compact type. An essential tool for this process is the Iwasawa decomposition of the Lie algebra of the connected component of the identity of the isometry group of G/K . Therefore, this might constitute a first step in order to produce more examples of isoparametric hypersurfaces in symmetric spaces of non-compact type than those presented in [2] as canonical extensions of isoparametric hypersurfaces of non-compact symmetric of rank one, and the homogeneous examples classified in [1].

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Harnack inequalities for evolving hypersurfaces – Julian Scheuer

Joint work with Paul Bryan and Mohammad N. Ivaki.

Let N^{n+1} be a Riemannian or Lorentzian manifold. Let a smooth, time-dependent family of embeddings $x : (0, T) \times M^n \rightarrow N^{n+1}$ of a closed, orientable and spacelike manifold M^n move according to the *curvature flow*

$$\dot{x} = -\sigma F\nu, \tag{1}$$

where F depends monotonically on the Weingarten operator of the hypersurface $M_t = x(t, M)$ and ν is a normal vector field along M_t with signature σ .

If the flow hypersurfaces are strictly convex, we introduce a new method to obtain so-called *differential Harnack inequalities* of Li-Yau-type, compare [4], of the form

$$\partial_t F - b(\nabla F, \nabla F) + \frac{p}{(p+1)t} F \geq 0,$$

where $p \neq 0$ is the homogeneity of F and b is the inverse of the second fundamental form. Until today, such estimates are known mostly for flows in the Euclidean space, for example for the mean curvature flow [3] and more general speeds [1]. In the sphere we obtained such estimates for a broad class of speeds [2]. If the flow is viewed in the form (1), the necessary computations are quite heavy. In [1] Andrews has used a reparametrisation by the Gauss map of a strictly convex hypersurface, which simplified the calculations tremendously. However such a map is generally not available in other manifolds.

In this talk we present a substitute for the reparametrisation of (1) by Gauss map, which simplifies the involved calculations in any Riemannian or Lorentzian manifold. We recover (and extend) the known results in the Euclidean case and in the sphere and obtain new Harnack inequalities for the mean curvature flow in locally symmetric Einstein manifolds and for more general flows in the De Sitter space. Via a duality method we obtain so-called *pseudo Harnack inequalities* for flows in the hyperbolic space, which until today tenaciously refused to allow the derivation of such results.

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Topology of dynamics of a nonhomogeneous rotationally symmetric ellipsoid on a smooth plane – Georgii Sechkin

Let us consider an ellipsoid of revolution moving on a smooth horizontal plane under the action of gravity. We construct topological invariants for this system and classify corresponding Liouville foliations up to Liouville equivalence. Two systems are called equivalent if they have the same closure of integral trajectories of systems solutions. Suppose that the mass distribution in the ellipsoid is such that it has an axis of dynamical symmetry coinciding with the axis of geometric symmetry. Moments of inertia about principal axes of inertia perpendicular to symmetry axis are equal to each other. We also assume that the center of mass lies on this symmetry axis (as in the Lagrange top) at distance s from the geometric center of the body.

A free rigid body has six degrees of freedom. We need three coordinates to describe the position of an arbitrary point in the body (e.g., the center of mass) with respect to a fixed space frame, and three more coordinates to describe the orientation of principal axes.

In our case, there is one holonomic constraint: the height of the center of mass above the plane is determined by the orientation of principal axes. Thus, the number of degrees of freedom is reduced to five. Let us write the equation in Eulers form using $f' = \{f, H\}$, where H is the Hamiltonian, and $\{, \}$ is the Poisson bracket on $e(3)^*$. Then in standard (S, R) coordinates we get the following first integrals: $H = \frac{1}{2} - \sum \frac{S_i^2}{A_i} + U$, where U is the potential energy and A is a constant, and $K = S_3$.

Using the Fomenko-Zieshang invariants, we prove the following theorem.

Theorem. The Liouville foliation associated with the above-described problem can be embedded in the foliation corresponding to the Zhukovsky system describing a heavy gyrostat.

Note that N. E. Zhukovsky (1899) found a generalization of Eulers integrable case, with Hamiltonian $H = \frac{1}{2} - \sum \frac{(S_i + \lambda_i)^2}{A_i}$. The additional integral is the same as in the Eulers case: $K = S_1^2 + S_2^2 + S_3^2$.

On the biconservative surfaces in Euclidean spaces – Rüyâ Yeğın Şen and Nurettin Cenk Turgay

An isometric immersion $f : M^m \rightarrow N^n$ between two Riemannian manifolds is called *biconservative* if its stress-energy tensor S_2 is conservative, i.e., $\tau_2(f)^\tau = 0$, where $\tau_2(f)$ is the bitension field of f . By considering the definition of τ_2 , it is obtained that f is biconservative if and only

if

$$m\nabla\|H\|^2 + 4\operatorname{trace}A_{\nabla^\perp H}(\cdot) + 4\operatorname{trace}(\tilde{R}(\cdot, H)\cdot)^T = 0,$$

where H , A and ∇^\perp are the mean curvature, shape operator and normal connection of M^m and \tilde{R} is the curvature tensor of N^n .

In this talk, we would like to present our recent results on the biconservative surfaces in Euclidean spaces. We consider biconservative surfaces with parallel normalized mean curvature vector in Euclidean spaces.

2000 MSC Codes. 53C42

Keywords: Biconservative surfaces, parallel normalized mean curvature vector

Compact Ricci solitons in dimension two – Udo Simon

The following result is known:

Any compact Ricci 2-soliton has constant Gaußcurvature.

For this result we give a new proof: we apply the index method to elliptic PDE systems in a version from Springer Lecture Notes 335.

Integrable complex structures on some 6-dimensional Lie algebras – Marcin Sroka

A complex structure on a Lie algebra \mathfrak{h} is an endomorphism $\mathcal{J} : \mathfrak{h} \rightarrow \mathfrak{h}$ such that $\mathcal{J}^2 = -id$. It is said to be integrable if

$$N(v, w) = -[v, w] + [\mathcal{J}v, \mathcal{J}w] - \mathcal{J}[\mathcal{J}v, w] - \mathcal{J}[v, \mathcal{J}w] = 0$$

for all $v, w \in \mathfrak{h}$, cf. [11]. The problem of determining whether a given Lie algebra admits an integrable complex structure or not is a well established and not an easy one, cf. [11]. During the talk we would like to present the recent progress in that subject, cf. [9, 1, 3, 5]. Just for the motivation, complex structures on a Lie algebra \mathfrak{h} correspond to left invariant almost complex structures on an associated Lie group H , what is more, the integrability of the first one is equivalent to the integrability of the second one. More interestingly, in the case when \mathfrak{h} is nilpotent or solvable, (integrable) complex structures on \mathfrak{h} correspond to left invariant (integrable) almost complex structures on an assassinated nilmanifold or solvmanifold (quotient of an associated group by a cocompact lattice, if there is such). These manifolds form a very nice class from couple of reasons. One can obtain a lot of information about them from the Lie algebra of the group they are quotients of, for example a classical result state that the de Rham cohomology of them is just the Lie algebra cohomology, cf. [6], the same is true for the Dolbeault cohomology of some nilmanifolds, cf. [4]. This is a huge simplification in a contrast with a completely abstract manifold. They provide also a great class of examples, nonexamples and testing objects for many conjectures, cf. [2], recall just a famous Kodaira-Turston nilmanifold being the example of a compact symplectic but nonKähler manifold. From these reasons the problem of classifying which Lie algebras admit an integrable complex structure is not only interesting in it's own right but has also a profound impact on our knowledge on many related objects.

First general results on existence of integrable complex structures on Lie algebras are probably due to Samelson, cf. [10], he proved that any compact Lie algebra, here it means having a compact associated Lie group, admits such a structure. Later it was proved by Morimoto, cf. [11], that the same is true for reductive Lie algebras. Going into the classification by dimensions we have that the problem is trivial for the real dimension two since then any complex structure is integrable. For the dimension four the problem was solved by Snow, cf. [12], and Ovando, cf. [7], for a solvable algebras. With a little work one can extend their results to a full classification in the dimension four since there are very few four dimensional nonsolvable Lie algebras, cf. [8]. In

the dimension six the problem is much harder and there are couple reasons for that. Firstly, the computational complexity is much bigger. Secondly, there is no classification of six dimensional Lie algebras, although the classification is know up to dimension five, cf. [8], which causes a lot of problems since most attempts to the question of existence of integrable complex structures rely on case by case study. The classification in dimension six is known for special complex structures, so called abelian complex structures, cf. [1], and for nilpotent algebras, cf. [9]. The work of Salamon was recently refined in [3] by computing moduli spaces of integrable complex structures for six dimensional nilpotent algebras. In [5] we have classified which Lie algebras of the form $\mathfrak{g} \times \mathfrak{g}$, for a three dimensional Lie algebra \mathfrak{g} , admit an integrable complex structure. This is a step toward a classification in dimension six at least for decomposable algebras.

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On infinitesimal harmonic transformations of complete Riemannian manifolds – Sergey Stepanov and Irina Tsyganok

A vector field X on a Riemannian manifold (M, g) is called an *infinitesimal harmonic transformation* of (M, g) if X generates a flow which is a local one-parameter group of harmonic diffeomorphisms (see [1]). We recall that the kinetic energy $E(X)$ of the flow generated by X (see [2, p. 2]) is determined by the following equation $E(X) = 2^{-1} \int_M \|X\|^2 d\text{Vol}_g$. Then we have the following

Theorem. *Let (M, g) be a complete Riemannian manifold (M, g) with nonpositive Ricci curvature. Then all possible infinitesimal harmonic transformations with finite kinetic energy are parallel. If the volume of (M, g) is infinite or the Ricci curvature is negative at some point, then every infinitesimal harmonic transformation with finite kinetic energy is identically zero.*

If g is a complete Riemannian metric, X is a vector field, and λ is a constant value on a manifold, then (g, X, λ) is called a *Ricci soliton* if the Ricci tensor Ric of the metric g satisfies the equation $-2\text{Ric} = L_X g + 2\lambda g$. In [3], it was shown that a vector field X of the Ricci soliton (g, X, λ) is

an infinitesimal harmonic transformation of a Riemannian manifold (M, g) . Then the following corollary holds.

Corollary. *Let (g, X, λ) be a Ricci soliton with complete Riemannian metric g on a manifold. If the Ricci curvature of g is nonpositive and the kinetic energy of the flow generated by X is finite, then g is a Ricci-flat metric. If, in addition, the volume of M which is defined by g is infinite then X is identically zero.*

These two statements are complementary to our results in [5].

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Affine hyperspheres with a \tilde{J} -tangent Blaschke field – Zuzanna Szancer

We study real affine hyperspheres $f : M \rightarrow \mathbb{R}^{2n+2}$ with a \tilde{J} -tangent affine normal field, where \tilde{J} is the canonical para-complex structure on \mathbb{R}^{2n+2} . We give a local classification of \tilde{J} -tangent affine hyperspheres of arbitrary dimension with an involutive distribution \mathcal{D} . Several examples are also given.

On Riemannian fat associated bundles – Anna Szczepkowska

Fat associated bundles constitute an important tool in constructing Riemannian metrics of positive and non-negative curvature. The so called fatness condition in general is rather complicated and thus was analyzed only in particular cases of associated bundles. I will remind classical results in the matter and introduce recent developments since we were able to find necessary conditions for the existence of such bundles in the case of arbitrary G -structures over homogeneous spaces. These conditions yield a kind of classification of fat bundles associated with G -structures over compact homogeneous spaces provided that the connection in a G -structure is canonical.

Diameter Bounds, Gap Theorems, and Hitchin-Thorpe Inequalities for Compact Quasi-Einstein Manifolds – Homare Tadano

In this poster, stimulated by Fernández-López and García-Río, we will give some lower diameter bounds for non-trivial compact shrinking quasi-Einstein manifolds in terms of the Ricci and scalar curvatures. As an application, we will give a gap theorem for compact shrinking quasi-Einstein manifolds by showing a necessary and sufficient condition for the manifolds to be Einstein. Moreover, following recent work by Ma and by Brasil, Costa, and Ribeiro Jr., we will provide a sufficient condition for four-dimensional compact shrinking quasi-Einstein manifolds to satisfy the Hitchin-Thorpe inequality, and to be isometric to the standard four-dimensional sphere \mathbb{S}^4 . Finally, using a classification of compact shrinking quasi-Einstein manifolds by Catino, we will prove that under a simple condition, a four-dimensional compact shrinking quasi-Einstein manifold with harmonic self-dual part of the Weyl tensor must be isometric to the four-dimensional

sphere \mathbb{S}^4 or the complex projective surface $\mathbb{C}\mathbb{P}^2$ with their canonical metrics, or must be Kähler-Einstein, or its universal covering must be globally conformally equivalent to the standard four-dimensional sphere \mathbb{S}^4 .

Some Cheeger-Gromov-Taylor Type Compactness Theorems for Ricci Solitons – Homare Tadano

An important problem in Riemannian geometry is to investigate the relation between topology and geometric structure on Riemannian manifolds. The celebrated theorem of S. B. Myers [8] guarantees the compactness of a complete Riemannian manifold under some positive lower bound on the Ricci curvature. This theorem can be considered as a topological obstruction for a complete Riemannian manifold to have a positive lower bound on the Ricci curvature. On the other hand, J. Lohkamp [6] proved that in dimension at least three, any manifold admits a complete Riemannian metric with negative Ricci curvature. Hence, in dimension at least three, there are no topological obstructions to the existence of a complete Riemannian metric with negative Ricci curvature. To give an interesting compactness criterion for complete Riemannian manifolds is one of the most important problems in Riemannian geometry, and the Myers theorem has been widely generalized in various directions by many authors.

The aim of this talk is to discuss the compactness of complete Ricci solitons. Ricci solitons were introduced by R. Hamilton in 1982 and are natural generalizations of Einstein manifolds. They correspond to self-similar solutions to the Ricci flow and often arise as singularity models of the flow. The importance of Ricci solitons was demonstrated by G. Perelman, where Ricci solitons played crucial roles in his affirmative resolution of the Poincaré conjecture. In this talk, after we reviewed basic facts on Ricci solitons, we will establish some new compactness theorems for complete shrinking Ricci solitons. Our results can be regarded as natural generalizations of the compactness theorem due to J. Cheeger, M. Gromov, and M. Taylor [1] and improve previous compactness theorems obtained by M. Fernández-López and E. García-Río [2], M. Limoncu [4, 5], Z. Qian [9], and G. Wei and W. Wylie [10].

If time permits, after making a brief review of Sasaki geometry, we will give a new compactness theorem for complete Sasaki manifolds. This result can also be regarded as a natural generalization of the compactness theorem due to J. Cheeger, M. Gromov, and M. Taylor [1] and improves Myers type theorems due to I. Hasegawa and M. Seino [3], and Y. Nitta [7].

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On Dupin hypersurfaces with constant Laguerre curvature – Keti Tenenblat

Proper Dupin hypersurfaces M^n in R^{n+1} , $n \geq 3$, parametrized by lines of curvature, and n distinct principal curvatures, will be considered. Known results on submanifolds M^n , with constant Moebius curvature, will be reviewed. Assuming that M^n that the principal curvatures do not vanish, all such Dupin hypersurfaces with constant Laguerre curvatures will be given explicitly. In particular, it will be shown that they are determined by n real constants, namely, $(n - 2)$ Laguerre curvatures and two other constants, one of them being nonzero.

Biconservative submanifolds with higher codimension in Riemannian space-forms – Nurettin Cenk Turgay

An isometric immersion $f : M \rightarrow N$ between two Riemannian manifolds is biconservative if the tangent component of $\tau_2(f)$ vanishes identically, where τ_2 is the bitension field of M in N . By considering the definition of τ_2 , one can see that this condition is equivalent to

$$m\nabla\|H\|^2 + 4\text{trace}A_{\nabla^\perp H}(\cdot) + 4\text{trace}(\tilde{R}(\cdot, H)\cdot)^T = 0,$$

where H , A and ∇^\perp are the mean curvature, shape operator and normal connection of f and \tilde{R} is the curvature tensor of N .

In this talk, we would like to present a survey our recent results on biconservative submanifolds. We consider submanifolds with higher codimension in the Riemannian space-form S^n and in the product space $S^n \times \mathbb{R}$.

2000 MSC Codes. Primary 53C50; Secondary 53C42

Keywords: Biconservative submanifolds, The product space $S^n \times \mathbb{R}$, Riemannian space-forms

Special Transformations on Weyl Manifold with a special metric connection – Mustafa Deniz Türkoğlu , F. Özdemir

In this work, we obtain conformal, and projective curvatures. Also, we examine the geometric structures, and the curvature properties of Weyl manifold with semi-symmetric recurrent metric connection under conformal and projective transformations.

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Riemann flows and solitons – Constantin Udriste and Ali Sapeeh Rasheed

The Riemann flow, as a new evolution problem in Riemannian geometry, was described recently in our papers [1], [2]. This paper is dedicated to posynomial Riemann solitons and their simulation by Maple tools.

AMS Subject Classification: 58J35, 58J45, 53C44.

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Quasi-Einstein metrics and affine structures – Xabier Valle-Regueiro

(Joint work with M. Brozos-Vázquez, E. García-Río and P. Gilkey)

A pseudo-Riemannian manifold (M, g) of dimension n is called quasi-Einstein if there is a solution $f \in C^\infty(M)$ of the equation

$$\text{Hes}_f + \rho - \mu df \otimes df = \lambda g \quad (1)$$

for some μ and $\lambda \in \mathbb{R}$, where Hes_f and ρ denote the Hessian of f and the Ricci tensor, respectively [3, 4]. This class of manifolds includes, as particular cases, Einstein metrics and gradient Ricci solitons (see [1]). For $\lambda = 0$, Equation (1) has a corresponding equation in affine geometry. Let (Σ, D) be an affine manifold with torsion-free connection D , and set

$$\text{Hes}_{\hat{f}}^D + 2\rho_{\text{sym}}^D - \mu d\hat{f} \otimes d\hat{f} = 0, \quad (2)$$

where $\text{Hes}_{\hat{f}}^D = Dd\hat{f}$ is the affine Hessian of \hat{f} and ρ_{sym}^D is the symmetric part of the Ricci tensor of (Σ, D) .

The two equations above are linked as follows [2]. If \hat{f} is a solution to Equation (2), then the cotangent bundle $T^*\Sigma$ of Σ equipped with a deformed Riemannian extension $g_{D,\Phi}$ is quasi-Einstein for $f = \pi^*\hat{f}$ and $\lambda = 0$, where $\pi : T^*\Sigma \rightarrow \Sigma$ is the projection. Conversely, a self-dual quasi-Einstein manifold which is not locally conformally flat is locally isometric to $(T^*\Sigma, g_{D,\Phi})$ and the potential function f is the pull-back to $T^*\Sigma$ of a solution of (2) in (Σ, D) .

The purpose of the talk is to detail the above correspondence and to show some affine properties related to the solutions of Equation (2) above.

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Lagrangian submanifolds in the homogeneous nearly Kähler $S^3 \times S^3$ related to classical differential equations – Luc Vrancken

Joint work with B. Bektaş, M. Moruz, J. Van der Veken.

Nearly Kähler manifolds have been studied intensively in the 1970's by Gray. These nearly Kähler manifolds are almost Hermitian manifolds for which the tensor field ∇J is skew-symmetric. In particular, the almost complex structure is non-integrable if the manifold is non-Kähler. A well known example is the nearly Kähler 6-dimensional sphere, whose complex structure J can be defined in terms of the vector cross product on \mathbb{R}^7 . Recently it has been shown by Butruille that

the only homogeneous 6-dimensional nearly Kähler manifolds are the nearly Kähler 6-sphere, the nearly Kähler $S^3 \times S^3$, the projective space $\mathbb{C}P^3$ and the flag manifold $SU(3)/U(1) \times U(1)$. All these spaces are compact 3-symmetric spaces.

There are two natural types of submanifolds of nearly Kähler (or more generally, almost Hermitian) manifolds, namely almost complex and totally real submanifolds. Totally real submanifolds are those for which the almost complex structure maps tangent vectors to normal vectors. A special case occurs when the dimension of the submanifold is half of the dimension of the ambient space. In that case such submanifolds are called Lagrangian and J interchanges the tangent and the normal space.

In this talk we study Lagrangian submanifolds of $S^3 \times S^3$. The first examples of such Lagrangian submanifolds were due to Schäfer and Smoczyk. Other examples have been recently discovered by Moroianu and Semmelmann. In this talk we study Lagrangian submanifolds for which the projection on the first component has nowhere maximal rank. We show that then the projection is necessarily a minimal surface. Conversely we show that starting from a minimal surface and any solution of another classical PDE we can construct a Lagrangian immersion.

Dynamics of Darboux curves on surfaces – Paweł G. Walczak

(Joint work with Ronaldo Garcia and Remi Langevin)

In 1872 [Da], Gaston Darboux defined a family of curves on surfaces in the 3-dimensional Euclidean space \mathbb{E}^3 which are preserved by the action of the Möbius group and share many properties with geodesics: A curve C on a surface S in \mathbb{E}^3 is called a Darboux curve whenever all the osculating spheres of C are tangent to S . We shall describe the generic behavior (“zig-zag” and “beak-to-beak”) of Darboux curves near ridge points of general surfaces and the dynamics of the Darboux curves on particular surfaces (canal surfaces, quadrics and certain cyclides).

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Symplectically fat bundles – Artur Woike

It is well known that there are two general ways to endow the total space of a fiber bundle with a fiberwise symplectic form. The first one is given by the Thurston’s theorem on symplectic fibrations. The second is given by the Sternberg, Weinstein and Lerman theorems on fat bundles and symplectic manifolds. We will call the symplectic fibrations constructed with the Sternberg and Weinstein theorem symplectically fat.

This talk is devoted to new constructions of symplectically fat fiber bundles. The latter are constructed in two ways: using the Kirwan map and expressing the fatness condition in terms of the isotropy representation related to the G -structure over some homogeneous space.

The examples of non-Keller mappings – Magdalena Woźniakowska

We consider the rare polynomial mappings of two complex variables having one and two zeros at infinity. We prove that if the Jacobian of these mappings is constant, it must be zero. The presentation concerns the problems related to the Keller mappings. Recall that the Keller mapping $F : C^2 \rightarrow C^2$ satisfies the condition $\text{Jac } F = \text{const} \neq 0$. In this presentation, non-Keller mappings are those whose the Jacobian being constant must vanish.

$\delta^\sharp(2, 2)$ -ideal hypersurfaces of dimension 5 in centroaffine differential geometry – Handan Yıldırım

Let M^n be an n -dimensional Riemannian manifold. Given integers $n \geq 3$ and $k \geq 1$, we denote by $\mathcal{S}(n, k)$ the finite set consisting of all k -tuples (n_1, \dots, n_k) of integers satisfying $2 \leq n_1, \dots, n_k < n$ and $n_1 + \dots + n_k \leq n$. Moreover, we denote by $\mathcal{S}(n)$ the union $\cup_{k \geq 1} \mathcal{S}(n, k)$. For each $(n_1, \dots, n_k) \in \mathcal{S}(n)$ and each $p \in M^n$, the invariant $\delta(n_1, \dots, n_k)(p)$ is defined by

$$\delta(n_1, \dots, n_k)(p) = \hat{\tau}(p) - \inf\{\hat{\tau}(L_1) + \dots + \hat{\tau}(L_k)\},$$

where $\hat{\tau}(p)$ is the scalar curvature of M^n at p , $\hat{\tau}(L_i)$ is the scalar curvature of L_i which is a subspace of $T_p M^n$ with $\dim L_i = n_i$, $i = 1, \dots, k$ and L_1, \dots, L_k run over all k mutually orthogonal subspaces of $T_p M^n$, ([2]). This invariant was used to determine an optimal lower bound for the mean curvature vector of the submanifolds of real space forms. Submanifolds attaining this bound are said to be ideal submanifolds.

Such a kind of invariant can be introduced in centroaffine differential geometry as follows:

$$\delta^\sharp(n_1, \dots, n_k)(p) = \hat{\tau}(p) - \sup\{\hat{\tau}(L_1) + \dots + \hat{\tau}(L_k)\}.$$

Theorem 0.1 *Let M^n be a definite centroaffine hypersurface of \mathbb{R}^{n+1} . Take $\epsilon = 1$ (respectively, $\epsilon = -1$) if M^n is positive (respectively, negative) definite. Then, for each k -tuple $(n_1, \dots, n_k) \in \mathcal{S}(n)$ with $n_1 + \dots + n_k < n$, we have*

$$\begin{aligned} \delta^\sharp(n_1, \dots, n_k) \geq & -\frac{n^2 \left(n - \sum_{i=1}^k n_i + 3k - 1 - 6 \sum_{i=1}^k \frac{1}{2+n_i} \right)}{2 \left(n - \sum_{i=1}^k n_i + 3k + 2 - 6 \sum_{i=1}^k \frac{1}{2+n_i} \right)} \|T^\sharp\|^2 \\ & + \frac{1}{2} \left(n(n-1) - \sum_{i=1}^k n_i(n_i-1) \right) \epsilon, \end{aligned}$$

where T^\sharp is Tchebychev vector field. The equality case of the above inequality holds at a point $p \in M^n$ if and only if one has

- $K_{BC}^A = 0$ if A, B, C are mutually different and not all in the same Δ_i with $i \in \{1, \dots, k\}$,
- $K_{\alpha_j \alpha_j}^{\alpha_i} = K_{rr}^{\alpha_i} = \sum_{\beta_i \in \Delta_i} K_{\beta_i \beta_i}^{\alpha_i} = 0$ for $i \neq j$,
- $K_{rr}^r = 3K_{ss}^r = (n_i + 2)K_{\alpha_i \alpha_i}^r$ for $r \neq s$.

Here, K is the difference tensor. Moreover, $\Delta_1 = (1, \dots, n_1)$, $\Delta_2 = (n_1 + 1, \dots, n_1 + n_2)$, ..., $\Delta_k = (n_1 + \dots + n_{k-1} + 1, \dots, n_1 + \dots + n_k)$, $\Delta_{k+1} = (n_1 + \dots + n_k + 1, \dots, n)$. Furthermore, $A, B, C \in \{1, \dots, n\}$, $i, j \in \{1, \dots, k\}$, $\alpha_i, \beta_i \in \Delta_i$ and $r, s \in \Delta_{k+1}$.

A centroaffine immersion of M^n into R^{n+1} is called $\delta^\sharp(n_1, \dots, n_k)$ -ideal if it satisfies the equality case of the above inequality identically. Moreover, it is called *ideal* if it is $\delta^\sharp(n_1, \dots, n_k)$ -ideal for the corresponding $(n_1, \dots, n_k) \in \mathcal{S}(n)$. So far, most results in this area have only been related with 3- and 4-dimensional $\delta^\sharp(2)$ -ideal centroaffine hypersurfaces (See [1]-[5]).

In this talk which is based on a joint work with Luc Vrancken in [6], we classify $\delta^\sharp(2, 2)$ -ideal hypersurfaces of dimension 5 in centroaffine differential geometry.

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Rigidity of Ricci Solitons with Weakly Harmonic Weyl Tensors – Gabjin Yun

A complete Riemannian metric g on a smooth manifold M^n is called a gradient Ricci soliton if there exist a constant ρ and a smooth function f on M satisfying

$$\text{Ric}_g + \text{Hess } f = \rho g \tag{1}$$

where Ric_g is the Ricci tensor of the metric g , and $\text{Hess } f$ denotes the Hessian of f . A gradient Ricci soliton satisfying (1) is said to be shrinking, steady or expanding according as $\rho > 0$, $\rho = 0$, or $\rho < 0$, respectively.

In this talk, we will show some rigidity results on gradient shrinking(or steady) Ricci solitons with weakly harmonic Weyl curvature tensor. We say that a gradient Ricci soliton (M, g) satisfying (1) has weakly harmonic Weyl curvature tensor if $\delta W(\cdot, \cdot, \nabla f) = 0$, where W denotes the Weyl curvature tensor.

First, we will show that if a compact gradient shrinking Ricci soliton (M, g) has weakly harmonic Weyl curvature tensor, then (M, g) is Einstein. In the case of noncompact, we will prove that if M is complete gradient shrinking Ricci soliton with weakly harmonic Weyl curvature tensor, then M is rigid in the sense that M is given by a quotient of product of an Einstein manifold with Euclidean space. These are generalizations of the previous known results. Finally, we will show that if (M^n, g) be a complete noncompact gradient steady Ricci soliton with weakly harmonic Weyl curvature tensor, and if the scalar curvature s_g attains its maximum at some interior point, then either (M, g) is flat or isometric to the Bryant soliton.

Metrics and connections critical for the total mixed scalar curvature of a distribution – Tomasz Zawadzki

The mixed scalar curvature of a distribution on a Riemannian manifold is the sum of sectional curvatures of pairwise orthogonal planes that intersect the distribution along a line. In this joint work with V. Rovenski we consider the total mixed scalar curvature of a fixed distribution on Riemannian and metric-affine manifolds, as a functional depending on a pseudo-Riemannian metric [1] or on a linear connection – analogously to the Einstein-Hilbert action. In each case, the Euler-Lagrange equations are formulated and some of their solutions are obtained.

In particular, metrics critical for the total mixed scalar curvature were found for codimension-one foliations, K-contact and 3-Sasakian manifolds. On the other hand, the existence of a critical linear connection depends on the geometric properties of the distribution with respect to the Levi-Civita connection.

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