Nonlinear (fractional) Schrödinger equations with sign-changing nonlinearities

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We look for ground state solutions to the following nonlinear (fractional) Schrödinger equation

$$(-\Delta)^{\alpha/2}u + V(x)u = f(x,u) - \Gamma(x)|u|^{q-2}u \text{ on } \mathbb{R}^N, \quad 0 < \alpha \leq 2,$$

where $V = V_{per} + V_{loc} \in L^\infty(\mathbb{R}^N)$ is the sum of a periodic potential V_{per} and a localized potential V_{loc}, $\Gamma \in L^\infty(\mathbb{R}^N)$ is periodic and $\Gamma(x) \geq 0$ for a.e. $x \in \mathbb{R}^N$ and $2 \leq q < 2^*_\alpha$. We assume that

$$\begin{cases} \inf \sigma(-\Delta + V) > 0, & \text{for } \alpha = 2, \\ \operatorname{ess\,inf} V(x) > 0, & \text{for } 0 < \alpha < 2, \end{cases}$$

where $\sigma(-\Delta + V)$ stands for the spectrum of $-\Delta + V$, and f has the subcritical growth but higher than $\Gamma(x)|u|^{q-2}u$, however the nonlinearity $f(x,u) - \Gamma(x)|u|^{q-2}u$ may change sign. Although a Nehari-type monotonicity condition for the nonlinearity is not satisfied we investigate the existence of ground state solutions being minimizers on the Nehari manifold. We also investigate the existence of infinitely many geometrically distinct solutions.

References

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