Periodic orbits near heteroclinics.

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Abstract

Let $W : \mathbb{R}^m \to \mathbb{R}$, $m \geq 1$ be a nonnegative potential with exactly two distinct zeros $a_\pm \in \mathbb{R}^m$. In the scalar case $m = 1$ phase plane analysis shows that the Newton equation

$$u'' = W_u(u) \tag{0.1}$$

possesses a heteroclinic solution u^∞ that connects a_- to a_+ and a family of periodic solutions u^T that converge in compacts to u^∞ as $T \to +\infty$.

We prove that, under the assumption that W is invariant under the reflection γ that exchange a_- to a_+, the same is true in the vector case $m > 1$.

We also extend this result to an infinite dimensional setting. We assume that W is invariant under a reflection σ that fixes $a_\pm$, $\sigma a_\pm = a_\pm$, and that there exist exactly two distinct heteroclinic solutions of (0.1) $\bar{u}_-$ and $\bar{u}_+$ that satisfy

$$\bar{u}_- = \sigma \bar{u}_+.$$

Under a non degeneracy assumption on $\bar{u}_\pm$ we show that, for $L > L_0$, the PDE system

$$\Delta u = W_u(u),$$

has a solution $u^L : \mathbb{R}^2 \to \mathbb{R}^m$ which is L-periodic in x, $u^L(x + L, y) = u^L(x, y)$, and such that the restriction of u^L to $(-\frac{L}{4}, \frac{L}{4}) \times \mathbb{R}$, converges along a subsequence to a heteroclinic connection $u^\infty : \mathbb{R}^2 \to \mathbb{R}^m$ between $\bar{u}_-$ and $\bar{u}_+$:

$$\lim_{L \to +\infty} u^L(x, \cdot) = u^\infty(x, \cdot),$$
$$\lim_{x \to \pm\infty} u^\infty(x, \cdot) = \bar{u}_\pm.$$