

Multiplicity results and qualitative properties for the Lane-Emden problem

Ianni Isabella



UNIVERSITÀ DEGLI STUDI DELLA CAMPANIA
LUIGI VANVITELLI

SCUOLA POLITECNICA E DELLE SCIENZE DI BASE

DIPARTIMENTO DI MATEMATICA
E FISICA

ABSTRACT. We consider the semilinear elliptic problem

$$\begin{cases} -\Delta u = |u|^{p-1}u & \text{in } B \\ u = 0 & \text{on } \partial B \end{cases} \quad (\mathcal{E}_p)$$

where B is the unit ball of \mathbb{R}^2 centered at the origin and $p \in (1, +\infty)$. We show the existence of new nonradial sign-changing solutions to (\mathcal{E}_p) which are *quasi-radial*, namely solutions whose nodal line is the union of a finite number of disjoint simple closed curves, which are the boundary of nested domains contained in B . In particular their nodal line doesn't touch ∂B . The result is obtained both via nonradial bifurcation from the least energy sign-changing radial solution of (\mathcal{E}_p) at certain values of p and by considering, for p large, the least energy nodal solutions in spaces of functions invariant by the action of the dihedral group generated by the reflection with respect to the x -axis and the rotation about the origin of angle $\frac{2\pi}{k}$, for suitable integers k .

We also show that for certain integers k the least energy nodal solutions in these spaces of symmetric functions are instead radial, exhibiting a breaking of symmetry phenomenon in dependence on the exponent p .