

RECENT RESULTS ON ALLEN-CAHN EQUATION: GLOBAL MINIMIZERS AND FINITE MORSE INDEX SOLUTIONS

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ABSTRACT. I will present two recent results on Allen-Cahn equation:

$$\Delta u + u - u^3 = 0$$

1) First we prove the existence of a global minimizer in dimension 8. (Joint with Y.Liu and K. Wang)

2) We prove that finite Morse index solutions to the Allen-Cahn equation in R^2

$$\Delta u + u - u^3 = 0$$

have **finitely many ends** and **linear energy growth**

$$\int_{B_R} |\nabla u|^2 + W(u) \leq C R$$

The main tool is a **curvature decay estimate** on level sets of these finite Morse index solutions, which in turn is reduced to a problem on the uniform second order regularity of clustering interfaces for the singularly perturbed Allen-Cahn equation. Using an indirect blow-up technique, inspired by the classical Colding-Minicozzi theory in minimal surfaces, we show that the obstruction to the uniform second order regularity is associated to the existence of nontrivial entire solutions to the **Toda system** in R^1

$$\Delta f_j = e^{-(f_{j+1}-f_j)} - e^{-(f_j-f_{j-1})}$$

For finite Morse index solutions in R^2 , we show that this obstruction does not exist by using information on stable solutions of the Toda system. The second order estimates are also extended to R^n ($n \geq 2$). (Joint work with Kelei Wang)

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